CMPUT 204 - Seminar Practice Question #2

Weeks of: September 23, 30 and October 7, 2013

Scope. Solving recurrence relations, analyzing recursive algorithms by constructing recurrence relations, counting arguments involving recurrence relations, designing simple algorithms

Problems selected from the following list will be discussed in the seminars, as time permits.

1. How do $n^2 + \sqrt{n} + 1$ and $22n^2 + 3n$ relate in terms of their asymptotic growth rate?
2. Prove or disprove: $2^n \in \Theta(3^n)$
3. Prove or disprove: for all functions f, g, $|f(n)| + |g(n)| \in \Omega(\min(|f(n)|, |g(n)|))$.
4. Solve the following recurrence relations:
   (a) $T(n) = T(n-1) + n$ for $n > 0$, and $T(0) = 0$
   (b) $T(n) = T(n/3) + 1$ for $n > 1$, and $T(1) = 1$. Assume $n = 3^k$.
5. Consider the following algorithm that computes the sum of the first n cubes: $S(n) = 1^3 + 2^3 + \cdots n^3$.

```
// assume n >= 0
function S(n)
if n <= 1 then
  return n
end
return S(n-1) + n*n*n
```

   (a) Set up and solve a recurrence relation for the number of calls of S for input n.
   (b) How does this algorithm compare to the straightforward iterative algorithm for computing the sum?

6. Consider the following recursive algorithm:

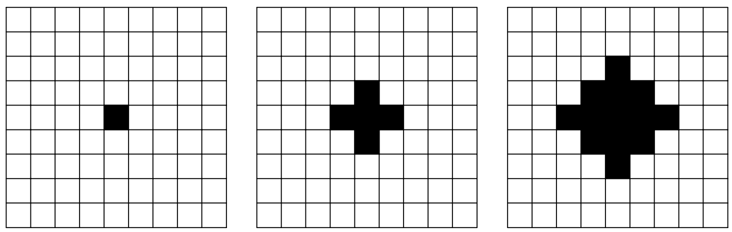
```
// assume n >= 1
function Q(n)
if n = 1 then
  return 1
end
return Q(n-1) + 2*n - 1
```

   (a) Set up and solve a recurrence relation for the functions return values to determine what this algorithm computes.
   (b) Set up and solve a recurrence relation for the number of multiplications.
   (c) Set up and solve a recurrence relation for the number of additions.

7. Problems 12 (Exercises 2.3) and Problem 12 (Exercises 2.4) in Levitin's textbook (on *von Neumann's neighbourhood*):

   (a) Consider the algorithm that starts with a single square and on each of its n iterations adds new squares all around the outside. How many one-by-one squares are there after n iterations? The result for $n = 0, 1$, and 2 are illustrated below.

   (b) Find the number of cells in the above part of range n by setting up and solving a recurrence relation.



$n = 0$ $n = 1$ $n = 2$