1. For the webnotes brute force set cover algorithm, for this input, what cover is found? What is its size? Give another cover of the same size.

```
subset  elements
      0 1 2 3 4 5 6
  0   - - - * - * -
  1   * - - * - * *
  2   - * - * - - -
  3   - - * - - * -
  4   - - * * - * *
  5   * * * - - * *
  6   * - - - - * *
  7   * - - - * * -
```

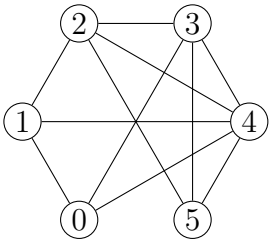
2. Is this CNF formula satisfiable? If yes, give a satisfying assignment; if no, explain why.

```
[ -1,  2, -4]
[ -1, -2,  4]
[ -1, -3, -4]
[ -1,  3,  4]
[ -1,  3, -4]
[  1,  2, -3]
[  1, -2,  4]
[  1, -2, -4]
[  1,  3,  4]
[  2,  3, -4]
```

   Recall -1 means $\neg x_1$, 2 means x_2, etc. So the above represents the formula

   $(\neg x_1 \vee x_2 \vee \neg x_4) \wedge \ldots \wedge (x_2 \vee x_3 \vee \neg x_4)$.

3. Find a largest clique in this graph.



4. Verify that (T, T, F, T, F, F) satisfies the CNF formula here. As a function of variables n and clauses m, give the runtime of your verification method.

```
[ -4, -5,  6]
[ -4,  5, -6]
[ -2,  4, -5]
[ -2,  5, -6]
[ -1, -3, -4]
[ -1, -3,  4]
[ -1,  4, -6]
[  1, -4,  5]
[  1, -3, -5]
[  1, -3,  4]
[  1,  5, -6]
[  2,  5,  6]
[  3, -5, -6]
[  4, -5,  6]
[  4,  5, -6]
[  4,  5,  6]
```

5. For each of the following: is the problem in NP? in P? Explain.

   clique Given a graph G and an integer k, does G have a k-clique?

   satisfiability Given a boolean formula f in CNF, is f satisfiable?

   distance Given a graph G, an integer k, and nodes a and b, is there a path between a and b with at most k edges?

   long path Given a graph G and an integer k, does G have a path (no repeated vertex) with at least k edges?

   set cover Given a set S of subsets of a set V and an integer k, is there a subset of at most k elements of S (each is a subset of V) that covers V?

   0-1 knapsack Given a set of items, their weights and values, a capacity C, and an integer k, is there a subset of items with total weight at most C and value at least k?

6. Give a version of the square root problem that is in NP.

1. Min cover found is {0,5,7}, size 3.

```
0  - - - * - * -
5  * * * - - * *
7  * - - - * * -
```

   Another min cover is {k,5,7}, where k is any of {1,2,4}. Another min cover is {2,4,7}.

2. yes. (T F T F).

3. {2,3,4,5} is a 4-clique. For each node x, remove x from the graph, and notice that the resulting graph is not a clique. So there is no 5-clique. So every 4-clique is a largest clique.

4. Verify that each clause is satisfied. E.g. the assignment has x_5 false, so the first clause $\neg x_4 \vee negx_5 \vee x_6$ is satisfied.

   Assuming you can access literals in constant time, and assignment values in constant time, the runtime is $\Theta(nm)$.

5. Each of the problems is in NP.

   Distance is in P. Each of the other problems is NP-complete. No NP-complete problem is known to be in P (and if it were, we would have P=NP, and whoever had proved this would be famous).

6. For an integer n, call t the integer root of n if $t * t \leq n$, and $(t+1) * (t+1) > n$.

   This problem is in NP:

   integer root Given an integer n and an integer t, is t the the integer root of n?

   What do you think? Is this problem in P?