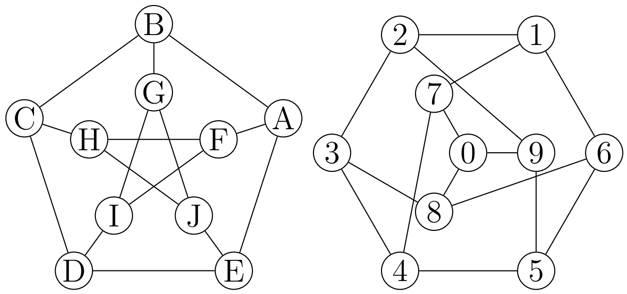
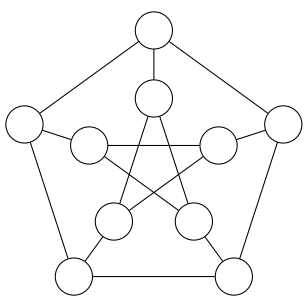
cmput 204 **seminar 8** **fall 2015**

1. For these graphs, give an isomorphism f that maps A to 0, B to 7, and E to 8. Then, give the 6-cycle in the 1st graph that corresponds to the 6-cycle (1,2,3,4,5,6).



Hint: edges have to map to edges, so paths have to map to paths, and cycles to cycles, etc. By considering 5-cycles, label the unlabelled graph with 0,...,9 so that it is equal to the second graph. Then give the isomorphism.



A	B	C	D	E	F	G	H	I	J
0	7			8					

2. Show the output from the brute-force 0-1 knapsack algorithm for the input below, with knapsack capacity 18.

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val   [7, 6, 10, 6, 9]
wt    [5, 8, 10, 9, 6]
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Repeat for capacity 23.

3. This proof is out of order. Rearrange it.

Claim. Let $(e_1, \ldots, e_{n-1})$ be the sequence of edges picked by an execution of Prim's algorithm. For each $j \in \{1, \ldots, n-1\}$, $S_j = \{e_1, \ldots, e_j\}$ is the subset of the edges of an MST.

Proof.

(a) So $w(c) = w(e_{t+1})$. So Z' is also a min spanning tree, and Z' contains $T_j \cup \{e_{j+1})$ (why ?).

(b) Also, T^+ has an edge of $C - e_{t+1}$ that crosses the cut defined by T (why?). Let c be such an edge with min cost.

(c) Assume that the claim holds for j up to t. We want to show that the claim holds for $j = t+1$. So, $T = \{e_1, \ldots, e_t\}$ is a subset of an MST Z. (why?)

(d) So T is a subset of Z, but $T \cup \{e_{t+1}\}$ is not. Let $T^+ = Z + \{e_{t+1}\}$. T^+ has a cycle C (why?).

(e) So we are done.

(f) and Z is a min spanning tree, so $Z \leq w(Z')$, so $w(c) \leq w(e_{t+1})$ (why ?).

(g) By induction on j. If $j = 0$ the claim holds. (why?)

(h) By Prim's algorithm, e_{t+1} is a min cost edge that crosses the cut defined by T, so $w(e_{t+1}) \leq w(c)$. But $Z' = Z + e_{t+1} - c$ is a spanning tree (why ?),

(i) If e_{t+1} is in Z we are done (why?). So assume e_{t+1} is not in Z.

1. (B,A,E,J,G) is a 5-cycle, so (7,0,8,x,y) must also be a 5-cycle. There are only two possibilities: x,y = 3,4, or x,y = 6,1. Continuing in this way, each of these two possibilities leads to an isomorphism. So there are two correct answers.

A	B	C	D	E	F	G	H	I	J
0	7	4	3	8	9	1	5	2	6
0	7	1	6	8	9	4	2	5	3

With the first isomorphism above, (G, I, D, C, H, J). With the second isomorphism above, (C, H, J, G, I, D).

2.
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(0,) 5 7
(2,) 10 10
(0, 1) 13 13
(0, 2) 15 17
(2, 4) 16 19
```

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(0,) 5 7
(2,) 10 10
(0, 1) 13 13
(0, 2) 15 17
(2, 4) 16 19
(0, 1, 2) 23 23
(0, 2, 4) 21 26
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3. (g)
(c)
(i)
(d)
(b)
(h)
(f)
(a)
(e)