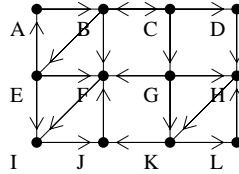1. In an undirected graph, the sum (over all vertices) of the degree of each vertex equals _____ times the number of edges.

   Repeat the question, replacing "undirected" with "directed".

2. On this digraph, trace the scc algorithm: (1) list nodes in postorder of transpose. Is the last node a sink? (2) using reverse order from (1), draw the dfs traversal forest (3) list the sc components.

3. **Claim: the last vertex in postorder of an acyclic digraph A is a source.**

   Assume (x,y) is an arc of A. (i) Assume dfs(x) is called before dfs(y). Prove that y appears before x in postorder.

   (ii) Repeat (i) if dfs(y) is called before dfs(x).

   (iii) Using (i) and (ii), prove the claim.

   Hint. With respect to the dfs traversal of a directed graph, define after(v) as the set of all vertices x with dfs(x) called after dfs(v).

   Observe: In the dfs forest, w is a descendant of v if and only if in the digraph, in the subgraph whose vertices are v and after(v), there is a dipath from v to w.

4. Give the runtime:

   ```
   def transpose(G):
     T = []
     for v in range(n(G)):
       nbrVec = []
       for w in range(n(G)):
         nbrVec.append(G[w][v])
       T.append(nbrVec)
     return T
   ```

   Repeat the question for the version of transpose that appears in the webnotes.

1. • two   • one

2. • C B E A D G I L H K J F Last node is not
   a sink, but it is in a sink scc.

   •

   ```
   F      K     G      D      A
   I      L                   B
   J      H                C   E
   ```

   • FIJ KLH G D ABCE

3. (i) dfs(y) is called after dfs(x), so y is in after(x).
   Also, there is a di-path (a single arc) from x to y.
   So, by the observation, in the dfs forest, y is in
   the subtree rooted at x. In postorder, all nodes
   of a subtree appear before the subtree root. So
   in postorder, y appears before x.

   (ii) x is in after(y). The only nodes in after(y)
   that appear before y in postorder are the de-
   scendants of y. But x is not a descendant of
   y (otherwise, there is a di-path from x to y (by
   the observation), which with the arc from y to x
   forms a dicycle, contradiction (since the digraph
   is acyclic)).

   (iii) Let z be the last vertex in postorder. By
   (i) and (ii), there is no arc from a vertex to z.
   So z is a source.

4. • There are n^2 iterations, so $\Theta(n^2)$.
   • $\Theta(n + \sum_v \deg(v)) = \Theta(n + m)$.