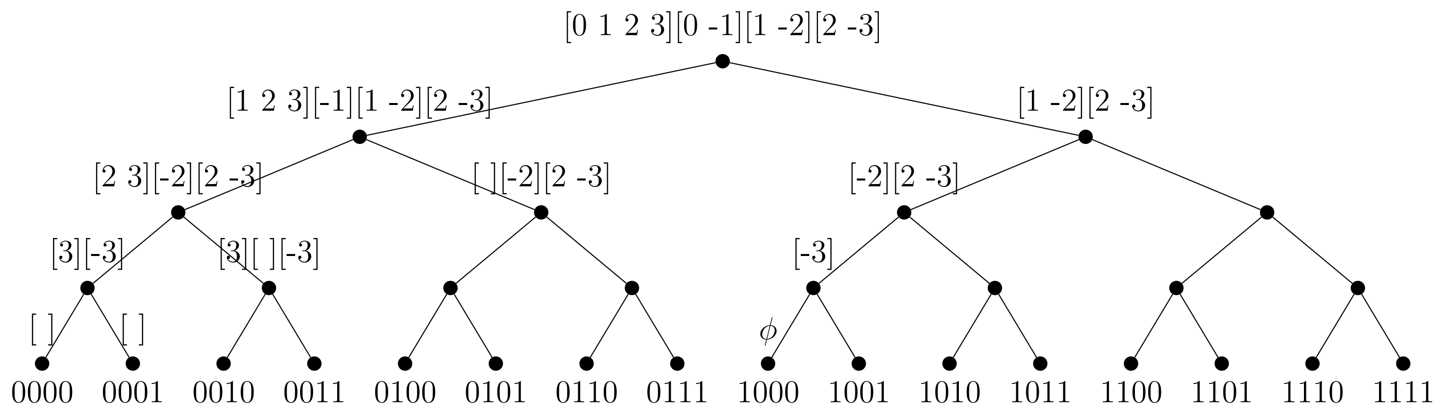
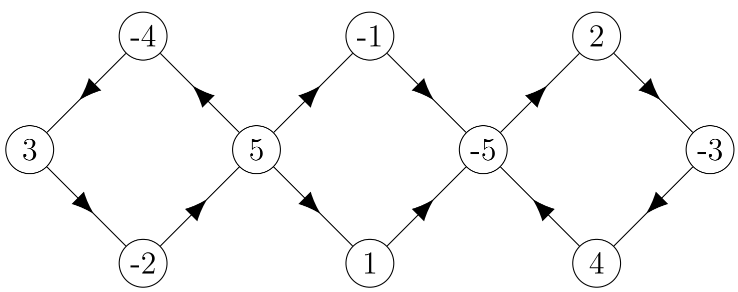
cmput 204 seminar 10 fall 2015



1. The above diagrams shows a backtrack search that stops once a satisfying assignment is found. (ii) Show the diagram if search continues until all satsifying assignments are found, and list all satisfying assignments.

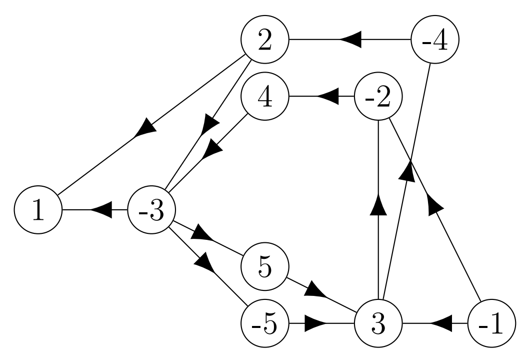
2. Trace DPLL sat solver on the formula `[0 1 2 3][0 -1][1 -2][2 -3]`.

3. Below is the implication digraph for f = `[[1,-5],[-2,-3],[3,4],[-4,-5],[2,5],[-1,-5]]`. Let $f' = f \wedge$ `[-2,-4]`. Draw the implication digraph for f', and explain whether f' is satisfiable.



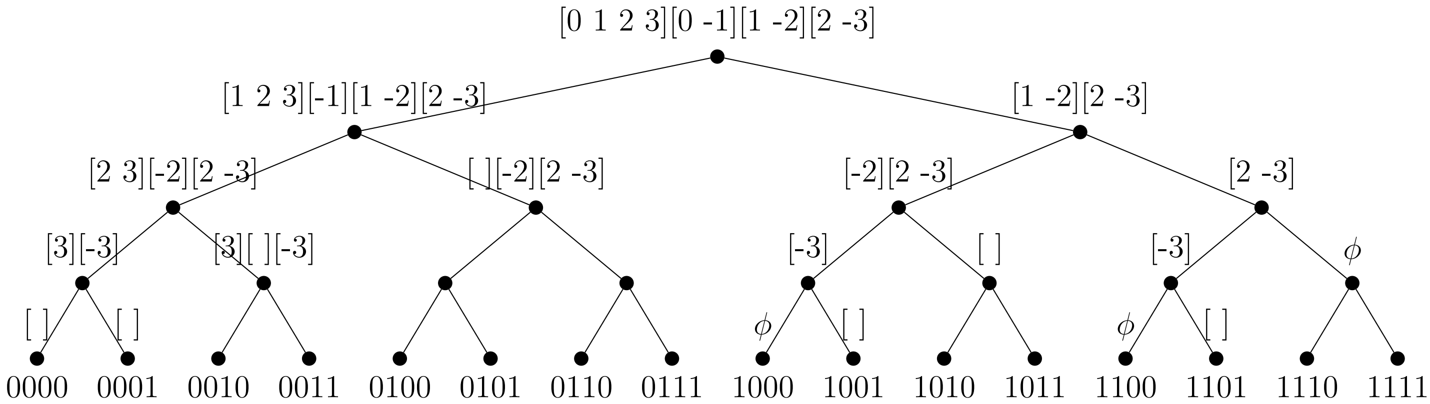
4. Below is the implication digraph for f = `[[1,-2],[1,3],[-2,-3],[2,4],`
`[-3,-4],[3,-5],[3,5]]`.

Let c = `[3,5]`, and let $f^- = f - c$, i.e. f with c removed. Draw the implication digraph for f^-, and explain whether f^- is satisfiable.



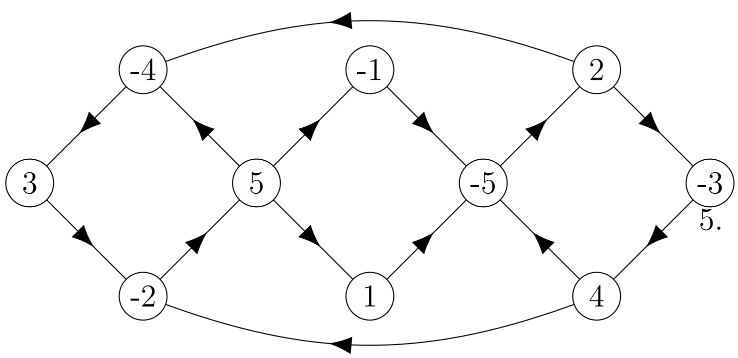
5. Explain carefully why 2sat is in P.

1. Assignments found are 1000, 1100, 111?.

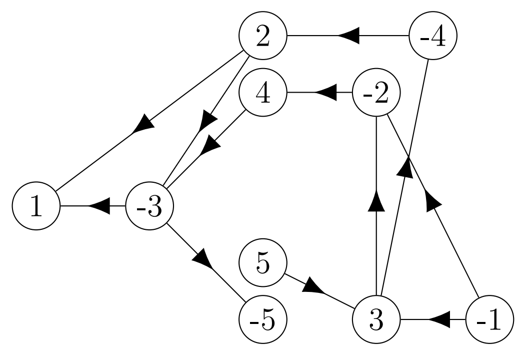


2. The formula has no unit clauses, but has pure literal x_0, so set x_0 true. Now the reduced formula is `[1 -2][2 -3]`. Again, there are no unit clauses, but two pure literals, x_1 and $\neg x_3$, so set one of these true, say $\neg x_3$. So set x_3 false. Now the reduced formula is `[1 -2]`. Again, there are no unit clauses, but two pure literals. Setting either one true satisfies the assignment.

3. unsatisfiable. There is a directed cycle (2 -4 3 -2 5 1 -5) that contains a literal x and its negation (here, x is 2 or 5), so there is no satisfying assignment.



4. satisfiable, since the digraph is acyclic. {-5} and {1} are sink sccs, so set 5 false and 1 true and remove from the digraph nodes {1,-1,5,-5}. Now {-3} is the only sink scc, so set 3 false and remove nodes {3,-3}. Now {2} and {4} are sink sccs, so set 2 true and 4 true, and we have a satisfying assignment 11010.



5. Assume n variables and m clauses. Assume there are no unit clauses (if there are, set the variable which satisfies that clause, remove that variable from the digraph, and continue), and there are no clauses `[x, -x]` (if there are, remove the clause). So each literal x can be in a clause with at most 2n-2 other literals, so m is at most $n(2n-2) \in \Theta(n^2)$.

   The implication digraph has $2n$ nodes and $2m \in \Theta(n^2)$ arcs. Finding sccs takes only linear time, so $\Theta(n+m) = O(n^2)$. Removing a sink scc can be done in linear time, and so can assigning the nodes from this scc, and updating the digraph. There are at most $2n$ updates, since each removes at least one node, and each update takes $O(n^2)$ time.

   So the total time is in $O(n^3)$.