last name ______________________ first names ______________________ ID ______________________

16oct2014 45min 30 marks no devices

1. [7 marks]

```
def ttt(G):
  seen = {}
  for v in G: seen[v] = False
  a,b,t = 0,[0],0
  for v in G:
    a+= 1
    if not seen[v]:
      t += 1
      ddd(G,v,seen,b)
  print a,b,t

def ddd(G,v,seen,b):
  print v,
  seen[v] = True
  for nbr in G[v]:
    b[0] += 1
    if not seen[nbr]:
      ddd(G,nbr,seen,b)
  print v,

G = {'A': [],
     'B': ['C', 'D', 'F'],
     'C': ['B', 'F'],
     'D': ['B', 'F'],
     'E': ['G'],
     'F': ['B', 'C', 'D'],
     'G': ['E'] }
```

- Draw G:

- Show the output from `ttt(G)`.

2. [4 marks] For integers $1 < j < n$, Euclid's `gcd(n,j)` performs $O($__________$)$ integer divisions, each on numbers with $O($__________$)$ bits. One such division takes $O($__________$)$ time, so the total runtime is $O($__________$)$.

3. [4 marks] Use binary numbers and Aryabhata's algorithm. Complete the work below: find the integer square root of 0b1011011101. **Circle** the square root. **Draw a box** around the remainder.

```
                1  1
              --------------
           \/ 10 11 01 11 01
1               1
                ----
                1 11
101             1 01
```

last name ______________ first names ______________ ID ______________

4. [6 marks]

```
def rmult(x,y):
  if y==0:
    print '(',x,y,')',0
    return 0
  elif 0==y%2: # y is even
    t = 2*rmult(x,y/2)
    print '(',x,y,')',t
    return t
  else:        # y is odd
    t = x+2*rmult(x,y/2)
    print '(',x,y,')',t
    return t
```

- Show the output from `rmult(25,25)`.

- Let $t = 9801$. Assume that, for all integers y with $0 \le y \le t$, `rmult(x,y)` returns x times y. Complete the proof below.

Claim. `rmult(x,9802)` returns x times 9802.

Proof. 9802 is even, so `rmult(x,9802)` returns ________________________. Also, __________ is less than __________,

so by our assumption ___________________ returns ___________________.

So ___________________________________

____

____

____

5. [3 marks] Consider the MillerRabin test to see whether 561 is prime. A number a is picked randomly from this set of integers: ______________________________. Next, a is raised to these powers (mod __________):

____.

Assume that $a^{560} = 1$ (mod __________). Then, by the ____________________ test, 561 is composite if ____.

6. [3 marks] In each case, give the simplest Θ expression for $T(n)$. Hint: master theorem: compare a and b^d.

- $T(n) = 9T(n/3) + \Theta(n^2)$

  a: b: d: $T(n) \in \Theta(\quad)$

- $T(n) = 6T(n/2) + \Theta(n^2)$

  a: b: d: $T(n) \in \Theta(\quad)$

- $T(n) = \Theta(n^3) + 8T(n/4)$

  a: b: d: $T(n) \in \Theta(\quad)$

7. [3 marks] Compute 5^{24} (mod 7). Show your work.