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cmput 204 **assignment 1** **due start of class, 2015 sep 21**

1. Write your answers on a copy of this document. Hand in 4 separate pages, not stapled. Write your name and id number on each page.

   For this assignment, your number $k = 11 + (n^{3191751} \pmod{89})$, where n is your student number. Write your student number and your number k. Check your answer, e.g. by using python `pow( )`.

   Acknowledge all sources; include all people with whom you discussed any part of this assignment (and for each source, list the relevant questions):

2. Let $t = k + 3700.09$. Using the Aryabhata longhand algorithm, find the first three digits in the square root of t. (So exactly one of your digits will be after the decimal place).

3. Let $j = k + 500$.

   (i) Using the decimal-binary conversion algorithm shown in class, convert j into binary.

   (ii) In binary, using the Aryabhata longhand algorithm, find the integer part of the square root of the number from (i).

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4.
```
def fib(n):
  if n<2: return n
  return fib(n-1) + fib(n-2)
```

(i) Assume that t is an integer such that `fib(t-2)` $\geq 1.6^{t-4}$ and `fib(t-1)` $\geq 1.6^{t-3}$.

Prove that `fib(t)` $\geq 1.6^{t-2}$.

(ii) Give the set of all integers q for which `fib(q)` $\geq 1.6^{q-2}$. Justify briefly.

(iii)

Does your answer to (ii) imply that `fib(n)` is in $O(1.6^n)$? Explain briefly.

Does your answer to (ii) imply that `fib(n)` is in $\Omega(1.6^n)$? Explain briefly.

Does your answer to (ii) imply that `fib(n)` is in $\Theta(1.6^n)$? Explain briefly.

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5.
```
def rmult(x,y): # x,y >= 0
  if y==0:      return 0
  elif 0== y%2: return 2*rmult(x, y/2)
  else:         return x + 2*rmult(x, y/2)
```

(i) Let $k' = 100 + k$, where k is your number. Draw the recursion tree for `rmult(k', 902)`. Beside each node in the tree, give the input parameters and the value returned.

(ii) Let $t = 9802$. Assume that, for all integers y with $0 \leq y \leq t$, `rmult(x,y)` returns the number equal to x times y. Prove that `rmult(x,9803)` returns the number equal to x times 9803.

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6. (i) How long does the usual school algorithm take to add two n-bit binary numbers, as a function of n? Explain briefly.

(ii) A certain algorithm takes cn^2 seconds for input size (i.e. number of bits) n. You run it with an input of size k. Then you run it with an input of size $2k$. What do you expect the ratio of the runtimes (second over first) to be? Justify briefly. Repeat the question if the time is cn^3.

(iii) As a function of n, give the runtime of `ifib(n)`. Justify briefly. Hint: it will be one of $\Theta(1)$, $\Theta(\lg n)$, $\Theta(n)$, $\Theta(n \lg n)$, $\Theta(n^2)$, $\Theta(n^2 \lg n)$, $\Theta(n^3)$, $\Theta(n^3 \lg n)$.

```
def ifib(n):
  a,b = 0,1
  for _ in range(n):
    a, b = b, a+b    #
  return a
```