

(i) two      (ii) one

(1) E H C A G F J D B I      (3) EH, C, AFGJ, BDI

Consider DFS traversal of a directed graph.

Define  $\text{after}(v)$  as the set of all vertices  $x$  with  $\text{DFS}(x)$  is called after  $\text{DFS}(v)$ .

**Observe** (exercise: prove by induction):

in the DFS forest, a node  $w$  is a descendant of node  $v$  if and only if in the digraph, in the subgraph whose vertices are  $v$  and  $\text{after}(v)$ , there is a dipath from  $v$  to  $w$ .

(i)  $\text{DFS}(y)$  is called after  $\text{DFS}(x)$ , so  $y$  is in  $\text{after}(x)$ . Also, there is a di-path (a single arc) from  $x$  to  $y$ . So, by the observation, in the DFS forest,  $y$  is in the subtree rooted at  $x$ . In postorder, all nodes of a subtree appear before the subtree root. So in postorder,  $y$  appears before  $x$ .

(ii)  $x$  is in  $\text{after}(y)$ . The only nodes in  $\text{after}(y)$  that appear before  $y$  in postorder are the descendants of  $y$ . But  $x$  is not a descendant of  $y$  (otherwise, there is a di-path from  $x$  to  $y$  (by the observation), which with the arc from  $y$  to  $x$  forms a dicycle, contradiction (since the digraph is acyclic)).

(iii) Let  $z$  be the last vertex in postorder. By (i) and (ii), there is no arc from a vertex to  $z$ . So  $z$  is a source.

$$\Theta(n + \sum_v \text{degree}(v)) = \Theta(n + m).$$

The for loop iterates exactly  $n^2$  times, so  $\Theta(n^2)$ .

```
def transpose(G):
    T = []
    for v in range(n(G)):
        nbrVec = []
        for w in range(n(G)):
            nbrVec.append(G[w][v])
        T.append(nbrVec)
    return T
```