

(i) In an undirected graph, the sum (over all vertices) of the degree of each vertex equals _____ times the number of edges.

(ii) In a directed graph, the sum (over all vertices) of the out-degree of each vertex equals _____ times the number of arcs.

Draw the digraph below. Then trace the scc algorithm on it: (1) list vertices in reverse postorder of the transpose (2) using the order from (1), draw the dfs traversal forest (3) list the sc components.

A: F, J B: D C: E D: G, I
 E: H F: G G: A, H H: E
 I: B, C, F J: C, F

Claim: the last vertex in postorder of an acyclic digraph A is a source.

(i) Assume that (x, y) is an arc of A . Assume $\text{dfs}(x)$ is called before $\text{dfs}(y)$. Prove that y occurs before x in postorder.

(ii) Repeat (i) if $\text{dfs}(y)$ is called before $\text{dfs}(x)$.

(iii) Using (i) and (ii), prove the claim.

Give the runtime:

```
def transpose(G):
    T = {}
    for v in G: T[v] = []
    for v in G:
        for w in G[v]: T[w].append(v)
    return T
```

Complete, and give the runtime:

```
def transpose(G):
    T = []
    for v in range(n(G)):
        nbrVec = []
        for w in range(n(G)):
            -----
            T.append(nbrVec)
    return T
```