win on week 1? Prob 1/3
win on week 2? Prob $2/3 * 1/3$
win on week 3? Prob $2/3 * 2/3 * 1/3$

expected number of weeks to win?
1∗ Pr(1st win week 1) + 2∗ Pr(1st win week 2) + 3∗ Pr(1st win week 3) . . .
$= \sum_{j=1}^{\infty} j * (2/3)^{j-1} * (1/3)$ So, how to evaluate this sum ?

This problem is so well known it has its own name: negative binomial distribution. Here is a short proof that the answer (here) is 3. Define E as the above sum. Notice that E satisfies this equation $E = 1 + 1/3 * 0 + 2/3 * E$. Why? because after week 1, you either don't play again (with probability 1/3), or you have to keep playing (with probability 2/3), in which case the expected number of times you have to play to win is still E. Now use this equation and solve for E.

By the law of the distribution of primes, the probability that a number at most x is prime is about $1/\ln(x)$. A number with 33 bits has value up to $n = 2^{33-1}$. So the probability that a number with at most 33 bits is prime is about, at least, $1/\ln(n) \approx 1/\ln(2^{33}) = 1/(33 \ln(2)) \approx 23$.

This is similar to the lottery problem. Here, the probability of winning the lottery is about 1/23, so the average number of trials before success is about 23.

About 1/2 of the numbers less than n are even, and only 1 is prime. So, the probability that an odd number less than n is prime is about two times the probability that any number less than n is prime. So, the expected number of trials will be about half that of the previous question, so about 11.5.