

1. $11^{**}23 \pmod{21} = 2 * 2 * 11 = 44 = 2$
 $11^{**}11 \pmod{21} = 2 * 2 * 11 = 44 = 2$
 $11^{**}5 \pmod{21} = 16 * 16 * 11 = (-5)(-5) \pmod{21} * 11 = 44 = 2$
 $11^{**}2 \pmod{21} = 11 * 11 = 121 = 121 - 105 = 16$
 $11^{**}1 \pmod{21} = 11$
2. $91 / 12 = 6, \text{ remainder } 19 = 7, \text{ remainder } 7$
 $45 / 12 = 2, \text{ remainder } 20+1 = 3, \text{ remainder } 9$
 $22 / 12 = 0, \text{ remainder } 22 = 1, \text{ remainder } 22-12 = 1, \text{ remainder } 10$
 $11 / 12 = 0, \text{ remainder } 11$
3. $\{ (\lg n)^5 \} \{ n^2 / ((\lg n)^3) \} \{ n^2 \lg n, n^2 \lg n + n^2 / (\lg n) \} \{ n^3, n^3 + \lg n \}$
 $\{ 2^{(n/3)} = (2^{1/3})^n \} \{ 2^{(n/2)} = (2^{.5})^n \} \{ n^2 2^{(n/2)} \} \{ 2^n, 2^n + 7n^3 + \lg n \}$
4. 0 []
1 [1]
2 [0, 1]
3 [1, 1]
4 [0, 0, 1]
5 [1, 0, 1]
6 [0, 1, 1]
7 [1, 1, 1]
8 [0, 0, 0, 1]
9 [1, 0, 0, 1]
5.

```
def modexp(a,e,n): # a>0 e>=0 n>=2
    L = qq(q(e))
    k , x = len(L), 1
    for j in range(k):
        if (0==(L[k-1-j]%2)): x = (x*x)%n
        else:                  x = (x*x*a)%n
    return x
```

0 1
1 2
2 4
3 8
4 16
5 32
6 64
7 28
8 56
9 12
10 24
11 48
...
17 72
18 44
19 88