

Unless stated otherwise, variables are integers.

1. By hand, compute  $11^{23} \bmod 21$ .
2. By hand, trace recursive binary division of  $91 \div 12$ . Give the quotient and remainder.
3. Sort the following functions into buckets. Two functions should be in the same bucket if and only if they have the same  $\Theta()$  complexity. Label the buckets with the simplest function that it contains (which may not be in the list below). Sort the buckets in increasing order.

$$\begin{array}{cccccc} (\lg n)^5 & 2^n & 2^n + 7n^3 + \lg n & n^3 + \lg n & n^2 \lg n + n^2 / (\lg n) \\ n^2 / ((\lg n)^3) & 2^{(n/2)} & n^2 2^{(n/2)} & 2^{(n/3)} & n^2 / \lg n \end{array}$$

4. Show the output.

```
def qq(n): # n >= 0
    L = []
    while (n>0):
        L.append(n%2)
        n = n/2
    return L
```

```
for j in range(10):
    print j, qq(j)
```

5. Fill in the blanks. Show the output.

```
def modexp(a,e,n): # a>=0 e>=0 n>=2
    L = qq(e)
    k , x = len(L), _____
    for j in range(k):
        if (0==(L[_____]%2)): x = _____
        else: x = _____
    return x
```

```
for j in range(11):
    print j, modexp(2,j,100)
```