2. one solution is to represent distances with an ordered pair (x, y), where x is the usual distance, and y is the number of edges. now $(a, b) < (c, d)$ if $a < c$, or if $a = c$ and $b = d$. you can use Dijkstra's algorithm with this new metric.

3. a) 19 b) 2 c)

edge included	cut partition
AE	ABCD EFGH
EF	ABCDE FGH
BE	AEFGH BCD
FG	ABE CDFGH
GH	ABEFG CDH
CG	ABEFGH CD
GD	ABCEFGH D

4. a)

vertex added	edge added	cost
A		0
B	AB	1
C	BC	3
G	CG	5
D	GD	6
F	GF	7
H	GH	8
E	AE	9

5. a) no: heaviest edge could be a bridge b) yes: remove e, add another edge from the same cycle c) yes: e belongs to the kruskal tree d) yes: otherwise some cycle joins the ends of e, so adding e and removing an edge of the cycle gives a lighter tree e) yes: consider the cut with the ends of e in different parts f) no: consider graph with edges ab, bc, cd, de, ef, fa, ad, with weights 1, 1, 1, 99, 99, 99, 10 g) no: consider a triangle (a,b,c) with edge weights ab 10, ac 11, bc 2 h) no: previous example i) yes j) yes: if the s-t path has an edge longer than r, then an edge of the r-path must be missing (else a cycle). add this missing edge, and remove the longer edge

6. (i) one way to do this: first prove that every mst is Kruskal (see class notes on web).

   next prove that if a weighted graph has a cycle C with a unique edge x of maximum weight on this cycle, then kruskal's algorithm will never pick this edge x. (argue by contradiction: suppose some kruskal execution X picks x. then, before x was picked, X had the choice of at least one edge on the cycle (because adding x does not create a cycle). the only reason the edge was not picked is because it would create a cycle. but this means, for every edge of $C - e$, the two ends of the edge are in the same component of the tree-so-far. but then the execution cannot pick x, because that would form a cycle, contradiction).

   now, to finish the proof: let e be any edge of a second mst that is not in the first mst T. adding e creates a cycle. e is picked by some execution of kruskal, so e cannot be the unique edge of max weight on this cycle, so there is some other edge on this cycle with the same weight as e.