solutions by text authors

- 2. a) A:1,14 B:15,16 C:2,13 D:3,10 E:11,12 F:4,9 G:5,6 H:7,8
 - b) sources A,B sinks G,H
 - c) algorithm outputs vertices in decreasing postorder, so B A C E D F H G

d) possible orderings are {A,B} C {D,E} F {G,H}, where order within each set is arbitrary, so total of $2 \times 2 \times 2 = 8$ orderings

- 3. i) first scc (say X) is CDFGHIJ, next (Y) is ABE; X is a source scc, Y is a sink SCC. metgraph has one arc, from X to Y. adding any arc from vertex in Y to vertex in X makes digraph sc. ii) sccs are found in order DFGHI (say X), C (Y), ABE (Z). X is sink, Z is source. adding any arc from vertex in X to vertex in Z makes digraph sc.
- 4. (b) only if: an odd cycle cannot be two-colored if: assume the input graph is not bipartite. let (x, y) be same-colored back edge, with x an ancestor of y. then the path length from x to y is even (same color); this path, with back-edge, gives odd cycle
- 5. true. there are two cases.

assume $\operatorname{pre}(u) < \operatorname{pre}(v) < \operatorname{post}(v) < \operatorname{post}(u)$. then u is ancestor of v. next assume $\operatorname{pre}(v) < \operatorname{post}(v) < \operatorname{pre}(u) < \operatorname{post}(u)$. then v was popped without considering u. But there is a (u, v) edge, and we consider all neighbors of v, contradiction.

- 6. e) if root has only one child, then it has degree one, so removing it leaves graph connected. dfs from first child explores every vertex reachable from a path avoiding the root. graph is undirected, so no edges from subtree of first child to subtree of any other child, so removing root disconnects the tree if the root has more than one child. f) if there is a backedge from a descdant of every child v' to an ancestor of v, each child can reach the entire tree above vso the graph is still connected after removing v. if there is a child v' such that none of its descendants have a backedge to an ancestor of v, then in the graph after removing v, there is no path between an ancestor of v and v'. (again, since the graph is undirected, there are no cross edges)
- 7. shortest path tree arcs: AB 1, BC 2, CD 1, CG 2, GF 1, GH 1, AE 4

	iteration										
	0	1	2	3	4	5	6	7			
А	0	0	0	0	0	0	0	0			
В	*	1	1	1	1	1	1	1			
С	*	*	3	3	3	3	3	3			
D	*	*	*	4	4	4	4	4			
Е	*	4	4	4	4	4	4	4			
F	*	8	7	7	7	7	6	6			
G	*	*	7	5	5	5	5	5			
Η	*	*	*	*	8	8	6	6			

8. shortest path tree arcs: SA 7, SE 6, EF -2, AB 4, AC -2, CD 2, BH -4, HG 1, GI -1,

iteration										
	0	1	2	3	4	5	6			
S	0	0	0	0	0	0	0			
А	*	7	7	7	7	7	7			
В	*	*	11	11	11	11	11			
С	*	6	5	5	5	5	5			
D	*	*	8	7	7	7	7			
Е	*	6	6	6	6	6	6			
F	*	5	4	4	4	4	4			
G	*	*	*	9	8	8	8			
Η	*	*	9	7	7	7	7			
Ι	*	*	*	*	8	7	7			