Computing Elo Ratings of Move Patterns in the Game of Go

Paper by Rémi Coulom, CG 2007

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Go Seminar, University of Alberta.

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Outline

Introduction

Minorization-Maximization / Bradley-Terry Models

Experiments in the Game of Go

Usage in a MC-Program

Conclusion
Introduction

- **Patterns** are useful for Go programs
  - Prune search trees
  - Order moves
  - Improve random simulations in Monte-Carlo programs

- One approach for learning patterns:
  Extract frequent patterns from expert games

- New supervised learning algorithm
  based on **Bradley-Terry model** (theoretical basis of Elo system)
Elo rating system

- Assign numerical strength value to players
- Compute strength from game results
- Estimates a probability distribution for future game results

Apply to move patterns

- Each move is a victory of one pattern over the others
- Elo ratings give a probability distribution over moves
Simplest approach: Measure frequency of play of each pattern
(Bouzy/Chaslot 2005) (Moyo Go Studio)

\[
\text{Rating(Pattern)} = \frac{\text{number of times played}}{\text{number of times present}}
\]

- Stronger patterns are played sooner → higher rating
- Does not take strength of competing patterns into account
  (Elo-rating analogy: measure only winning rate independent of opponent strength)
Bayesian pattern ranking
(Stern/Herbrich/Graepel 2006)

- Takes strength of opponents into account
- Patterns to evaluate grows exponentially with number of features
- Restricted to only a few move features

Maximum-entropy classification
(Araki/Yoshida/Tsuruoka/Tsujii 2007)

- Addresses the problem of combining move features
- Does not take strength of opponents into account
- High computational cost
Introduction

Minorization-Maximization / Bradley-Terry Models
- Elo Ratings and the Bradley-Terry Model
- Generalizations of the Bradley-Terry Model
- Relevance of the Bradley-Terry Model
- Bayesian Inference
- Minorization-Maximization

Experiments in the Game of Go

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Elo Ratings and the Bradley-Terry Model

\[ P(i \text{ beats } j) = \frac{\gamma_i}{\gamma_i + \gamma_j} \]

(Elo rating of \( i \) is defined by \( r_i = 400 \log_{10}(\gamma_i) \))
Generalizations of the Bradley-Terry Model

Competitions between more than one individual:

\[
\forall i \in \{1, \ldots, n\}, P\text{ (}i\text{ wins)} = \frac{\gamma_i}{\gamma_1 + \gamma_2 + \cdots + \gamma_n}
\]

Competitions between teams:

\[
P\text{ (}1\text{-}2\text{-}3 \text{ wins against } 4\text{-}2 \text{ and } 1\text{-}5\text{-}6\text{-}7) = \frac{\gamma_1\gamma_2\gamma_3}{\gamma_1\gamma_2\gamma_3 + \gamma_4\gamma_2 + \gamma_1\gamma_5\gamma_6\gamma_7}
\]

(Hunter 2004)
Relevance of the Bradley-Terry Model

- Strong assumptions about what is being modeled
- No cycles
- Strength of a team is the sum of its members (in Elo ratings)
The values $\gamma_i$ have to be estimated from past results $R$ using Bayesian inference:

$$P(\gamma|R) = \frac{P(R|\gamma)P(\gamma)}{P(R)}$$

- Find $\gamma^*$ that maximizes $P(\gamma|R)$
- Convenient way to choose a prior distribution $P(\gamma)$ by virtual game results $R'$: $P(\gamma) = P(R'|\gamma)$
  $\rightarrow$ maximize $P(R,R'|\gamma)$
Minorization-Maximization

Notation

- $n$ individuals with unknown strengths $\gamma_1, \ldots, \gamma_n$
- $N$ results $R_1, \ldots, R_N$
- Probability of one result $R_j$ as a function of $\gamma_i$:

\[ P(R_j) = \frac{A_{ij}\gamma_i + B_{ij}}{C_{ij}\gamma_i + D_{ij}} \]

$A_{ij}, B_{ij}, C_{ij}, D_{ij}$ do not depend on $\gamma_i$. Either $A_{ij}$ or $B_{ij}$ is 0.

- Objective to maximize:

\[ L(\gamma_i) = \prod_{j=1}^{N} P(R_j) \]
Minorization-Maximization

- Make initial guess $\gamma^0$
- Find function $m$ that minorizes $L$ at $\gamma^0$
  - $m(\gamma^0) = L(\gamma^0)$ $\forall \gamma : m(\gamma) \leq L(\gamma)$
- Compute maximum $\gamma^1$ of $m$
- $\gamma^1$ is an improvement over $\gamma^0$
Function to be maximized

\[ L(\gamma_i) = \prod_{j=1}^{N} \frac{A_{ij}\gamma_i + B_{ij}}{C_{ij}\gamma_i + D_{ij}} \]

Take logarithm:

\[ \log L(\gamma_i) = \sum_{j=1}^{N} \log(A_{ij}\gamma_i + B_{ij}) - \sum_{j=1}^{N} \log(C_{ij}\gamma_i + D_{ij}) \]

Define number of wins: \( W_i = |\{j| A_{ij} \neq 0\}| \)

Remove terms that do not depend on \( \gamma_i \)

\[ f(\gamma_i) = W_i \log \gamma_i - \sum_{j=1}^{N} \log(C_{ij}\gamma_i + D_{ij}) \]
Logarithms can be minorized by their tangent at $x_0$:

\[ 1 - \frac{x}{x_0} - \log x_0 \]

**Fig. 2.** Minorization of $-\log x$ at $x_0 = 0.5$ by its tangent.
Minorization-Maximization

Minorizing function to be maximized becomes:

\[ m(\gamma_i) = W_i \log \gamma_i - \sum_{j=1}^{N} \frac{C_{ij} \gamma_i}{C_{ij} \gamma_i + D_{ij}} \]

Maximum of \( m \) is at:

\[ \gamma_i = \frac{W_i}{\sum_{j=1}^{N} \frac{C_{ij}}{C_{ij} \gamma_i + D_{ij}}} \]
Minorization-Maximization Formula:

\[ \gamma_i \leftarrow \frac{W_i}{\sum_{j=1}^{N} \frac{C_{ij}}{C_{ij}\gamma_i + D_{ij}}} \]

- A win counts more if
  - team mates are weak \((C_{ij})\)
  - overall strength of participants is high \((C_{ij}\gamma_i + D_{ij})\)

- Updates can be done
  - one \(\gamma_i\) at a time
  - in batches (only for mutually exclusive features)
Experiments in the Game of Go

Introduction

Minorization-Maximization / Bradley-Terry Models

Experiments in the Game of Go

Data
Features
Prior
Results
Discussion

Usage in a MC-Program

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Computing Elo Ratings of Move Patterns
Each position of a game is a competition
The played move is the winner
Each move is a team of features
Introduction

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Experiments in the Game of Go

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Conclusion

Data

- Game records by strong players on the KGS Go server
- Either one player is 7d or stronger or both are 6d
- Training set: 652 games (131,939 moves)
- Test set: 551 games (115,832 moves)
Features

- **Tactical features**
  1. pass
  2. capture
  3. extension
  4. self-atari
  5. atari
  6. distance to border
  7. distance to previous move
  8. distance to move before previous move

- **Monte-Carlo owner** (63 random games)

- **Shape patterns**
  (16,780 shapes of radius 3–10 that occur at least 5000 times in training set)
Virtual opponent with $\gamma = 1$

Add one virtual win and one virtual loss against the virtual opponent for each feature

In Elo-rating, this corresponds to a symmetric probability distribution with mean 0 and standard deviation 302
## Results

<table>
<thead>
<tr>
<th>Feature</th>
<th>Level</th>
<th>$\gamma$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>1</td>
<td>0.17</td>
<td>Previous move is not a pass</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>24.37</td>
<td>Previous move is a pass</td>
</tr>
<tr>
<td>Capture</td>
<td>1</td>
<td>30.68</td>
<td>String contiguous to new string in atari</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.53</td>
<td>Re-capture previous move</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.88</td>
<td>Prevent connection to previous move</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.43</td>
<td>String not in a ladder</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.30</td>
<td>String in a ladder</td>
</tr>
<tr>
<td>Extension</td>
<td>1</td>
<td>11.37</td>
<td>New atari, not in a ladder</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.70</td>
<td>New atari, in a ladder</td>
</tr>
<tr>
<td>Self-atari</td>
<td>1</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Atari</td>
<td>1</td>
<td>1.58</td>
<td>Ladder atari</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10.24</td>
<td>Atari when there is a ko</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.70</td>
<td>Other atari</td>
</tr>
<tr>
<td>Distance to border</td>
<td>1</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.28</td>
<td></td>
</tr>
</tbody>
</table>
### Results

**Distance to previous move**

<table>
<thead>
<tr>
<th>Move</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.32</td>
</tr>
<tr>
<td>3</td>
<td>2.84</td>
</tr>
<tr>
<td>4</td>
<td>2.22</td>
</tr>
<tr>
<td>5</td>
<td>1.58</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>16</td>
<td>0.33</td>
</tr>
<tr>
<td>≥ 17</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Distance to the move before the previous move**

<table>
<thead>
<tr>
<th>Move</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.08</td>
</tr>
<tr>
<td>3</td>
<td>2.38</td>
</tr>
<tr>
<td>4</td>
<td>2.27</td>
</tr>
<tr>
<td>5</td>
<td>1.68</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>16</td>
<td>0.66</td>
</tr>
<tr>
<td>≥ 17</td>
<td>0.70</td>
</tr>
</tbody>
</table>

**MC Owner**

<table>
<thead>
<tr>
<th>MC Owner</th>
<th>Distance</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04</td>
<td>0 – 7</td>
</tr>
<tr>
<td>2</td>
<td>1.02</td>
<td>8 – 15</td>
</tr>
<tr>
<td>3</td>
<td>2.41</td>
<td>16 – 23</td>
</tr>
<tr>
<td>4</td>
<td>1.41</td>
<td>24 – 31</td>
</tr>
<tr>
<td>5</td>
<td>0.72</td>
<td>32 – 39</td>
</tr>
<tr>
<td>6</td>
<td>0.65</td>
<td>40 – 47</td>
</tr>
<tr>
<td>7</td>
<td>0.68</td>
<td>48 – 55</td>
</tr>
<tr>
<td>8</td>
<td>0.13</td>
<td>56 – 63</td>
</tr>
</tbody>
</table>

The distance between two points is defined as:

\[ d(\delta x, \delta y) = |\delta x| + |\delta y| + \max(|\delta x|, |\delta y|) \]
Mean log-evidence per game stage

- Mean logarithm of probability of selecting the target move
- Better in the middle and endgame, worse in the beginning
  (but Stern/Herbrich/Graepel used 12,000,000 shape patterns)

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Computing Elo Ratings of Move Patterns
Probability of finding the target move within $n$ best moves

- Minorization-Maximization
  - Stern, Herbrich, and Graepel (2006)
- Computing Elo Ratings of Move Patterns
Discussion

- **Best result** among results published in academic papers (De Groot (Moyo Go Studio) claims 42% not backed by publication)

- Used **much less games (652) and shape patterns (16,780)** than Stern/Herbrich/Graepel (181,000 games; 12,000,000 shape patterns)

- Training took only **1 hour CPU time** and 600 MB RAM
Usage in a MC-Program

Introduction

Minorization-Maximization / Bradley-Terry Models

Experiments in the Game of Go

Usage in a MC-Program
  Random Simulations
  Progressive Widening
  Performance against GNU Go

Conclusion
Random Simulations

- Patterns provide **probability distributions** for random games
- Only fast, **lightweight features**
  - $3 \times 3$ shapes
  - extension (without ladder knowledge)
  - capture (without ladder knowledge)
  - self-atari
  - contiguity to previous move
- **Contiguity to previous move** is a strong feature
  Produces sequences of contiguous moves like in MoGo

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Computing Elo Ratings of Move Patterns
Progressive Widening

- Crazy Stone uses patterns to prune the search tree
- Full set of features

1. Node in search tree is first searched for a while with random simulations
2. Then node is promoted to internal node and pruning is applied

Pruning algorithm:
Restrict search to first \( n \) node, with \( n \) growing with the logarithm of number of simulations:
add \( n^{\text{th}} \) node (\( n \geq 2 \)) after \( 40 \times 1.4^{n-2} \) simulations

- Due to strength of contiguity feature, this tends to produce a local search
Performance against GNU Go

<table>
<thead>
<tr>
<th>Pat.</th>
<th>P.W.</th>
<th>Size</th>
<th>Min./game</th>
<th>GNU Level</th>
<th>Komi</th>
<th>Games</th>
<th>Win ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>9 x 9</td>
<td>1.5</td>
<td>10</td>
<td>6.5</td>
<td>170</td>
<td>38.2%</td>
</tr>
<tr>
<td>x</td>
<td>-</td>
<td>9 x 9</td>
<td>1.5</td>
<td>10</td>
<td>6.5</td>
<td>170</td>
<td>68.2%</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>9 x 9</td>
<td>1.5</td>
<td>10</td>
<td>6.5</td>
<td>170</td>
<td>90.6%</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>19 x 19</td>
<td>32</td>
<td>8</td>
<td>6.5</td>
<td>192</td>
<td>0.0%</td>
</tr>
<tr>
<td>x</td>
<td>-</td>
<td>19 x 19</td>
<td>32</td>
<td>8</td>
<td>6.5</td>
<td>192</td>
<td>0.0%</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>19 x 19</td>
<td>32</td>
<td>8</td>
<td>6.5</td>
<td>192</td>
<td>37.5%</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>19 x 19</td>
<td>128</td>
<td>8</td>
<td>6.5</td>
<td>192</td>
<td>57.1%</td>
</tr>
</tbody>
</table>

**Table 2.** Match results. P.W. = progressive widening. Pat. = patterns in simulations.

- GNU Go 3.6
- Opteron 2.2 GHz:
  15,500 sim/sec (9 x 9), 3,700 sim/sec (19 x 19)
Conclusion / Future Work

- Generalized Bradley-Terry model is a powerful technique for pattern learning
  - simple and efficient
  - allows large number of features
  - produces probability distribution over legal moves for MC
- Principle of Monte Carlo features could be exploited more
- Validity of the model could be tested and improved:
  - Use only one (or few) sample per game to improve independence of samples
  - Test linearity hypothesis of Bradley-Terry model (strength of team is sum of strength of members)
  - Estimate the strength of some frequent feature pairs