Time Complexity and Intersection Graphs

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\{ \{2, 3, 4\}, \{2, 3\}, \{1, 2\}, \{5, 3\} \}
Which sets do I mean?

\{1, 2, 3, 4\}
Intervals on a Line

“Grandfather” of Intersection Classes

Figure from en.wikipedia.org
P and NP

NP Problems

P Problems

NP Complete
P and NP

- NP Problems
- P Problems
- NP Complete

YES
P and NP

NP Problems

P Problems

NP Complete

Problem in NP

Our Problem
P and NP

NP Problems

P Problems

NP Complete

Problem in \( \text{NP} \)

Our Problem
Recognition

In Class 🎯 G = (V, E) 🎯 Not In Class
Recognition

Sandwiching  Dichotomy
Sandwiching Reductions

R → Q

Yes

No
Sandwiching Reductions

R → Q

Yes → Yes
No → No

Diagram:

R

Q

Yes → Yes
No → No

Diagram:

R

Q

Yes → Yes
No → No

Diagram:

R

Q

Yes → Yes
No → No

Diagram:
Sandwiching Reductions

Instance of: Is it in Class A?

Instance of: Is it in Class B?

Yes

No
Sandwiching Reductions

$G$ is $k$-colourable

$G'$ is Class B $\iff$ $G'$ is Class A

Class A

Class B
Sandwiching Reductions

G is k-colourable

G' is Class B

G' is Class J

G' is Class A

Class A

Class J

Class B
Dichotomies

Overlap graphs of trees with $k \geq 3$ leaves

Intersection graphs of paths in trees with max degree $k \geq 3$

Overlap graphs of trees with $k < 3$ leaves

Intersection graphs of paths in trees with max degree $k < 3$
There's lots being done on set representations!

Interesting geometric sets
New classes
Polytime algorithms for otherwise NP-C problems
Applications
Minimizing representation size
Dynamic graph labelling
Understanding hardness
Approximability
Recognition problems
Graphical applications
General frameworks for intersection
The Helly property and set interaction
Thanks!
Intervals on a Line

Representational Savings

\[A = 1 - 13\]
\[C = 6 - 25\]
\[G = 28 - 31\]

How big? \(2n\)
Intervals on a Line
What happens on a line?
Intervals on a Line

Duke of Densmore
Intervals on a Line

Duke of Densmore
Coloring Unit Disc Graphs

Ad-hoc network modelling.

Figure from http://www.lab2.kuis.kyoto-u.ac.jp/~hashimoto/study/UnitDiskGraph.png
Two Big Questions

Given graph $G$, type of sets

1. Can $G$ be represented by that type of sets?
2. How small can we make the representing sets?
Representation Size

Every graph is an intersection graph and overlap graph.

How small can we make the union of the sets?

Easy to do in order of the edges.
Thanks!
Thanks!

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Graphs