Image statistics and Texture modeling

Dana Cobzas

[Srivastana, Simoncelli, Zhu – on the advances in statistical modelling of natural images IMIV 2003]

[Huang, Mumford – Statistics of Natural Images and Models, CVPR 1999]

[Mallat – a wavelet tour of signal processing, 1999]

Overview

- Spectral analysis
  - Fourier transform
  - Windowed Fourier Transform, Gabor filters
  - Structure tensor
- Connection to simple cells in primary visual cortex
- Image statistics
- Statistical models in image space
- Paper

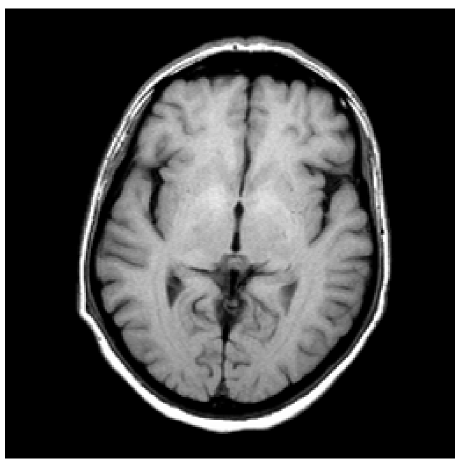
Fourier transform

Linear operator that decomposes a function into a continuos spectrum of frequency components

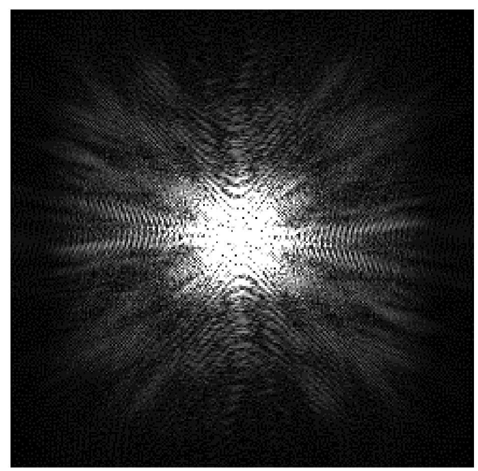
FT $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\omega) e^{i\omega t} d\omega.$$

Sinusoidal wave

IFT $$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt.$$



FT



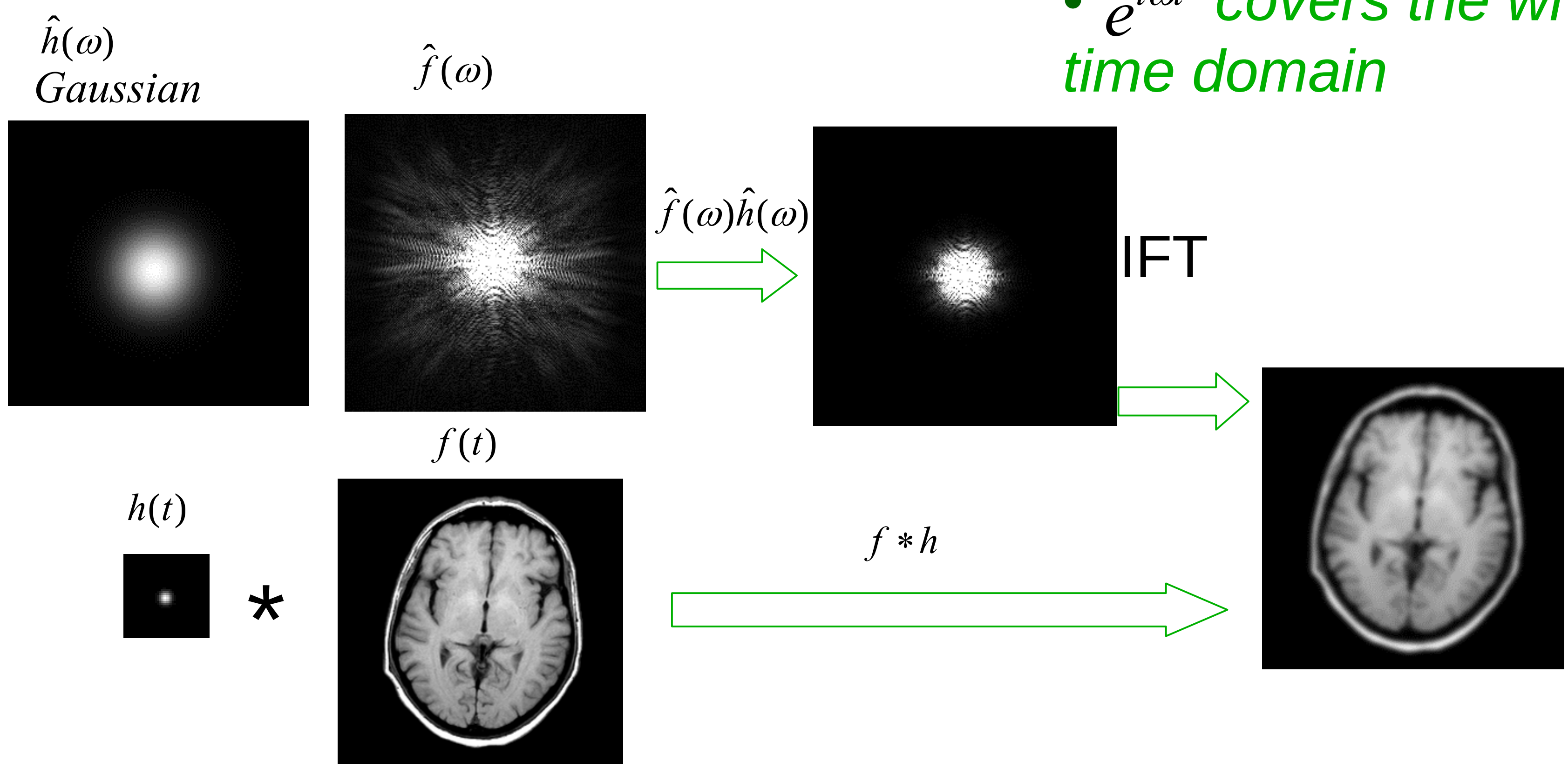
Spatial domain t Frequency domain ω

Filtering in frequency domain

Linear time invariant operator

$$Lf(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\omega)\hat{h}(\omega)e^{i\omega t}\,d\omega.$$

- *localized in frequency* *<u>but not in space</u>*
- $e^{i\omega t}$ *covers the whole time domain*



Windowed Fourier Transform

Gabor atoms

- time-frequency atoms that have a minimal spread in time-frequency plane
- Translation in time and frequency of a time window g

$$g_{u,\xi}(t) = g(t-u)\,e^{i\xi t}.$$

$$\hat{g}_{u,\xi}(\omega) = \hat{g}(\omega-\xi)\,e^{-iu(\omega-\xi)}.$$

concentrate energy in the
- *spatial neighborhood* u
- *frequency neighborhood of* ξ

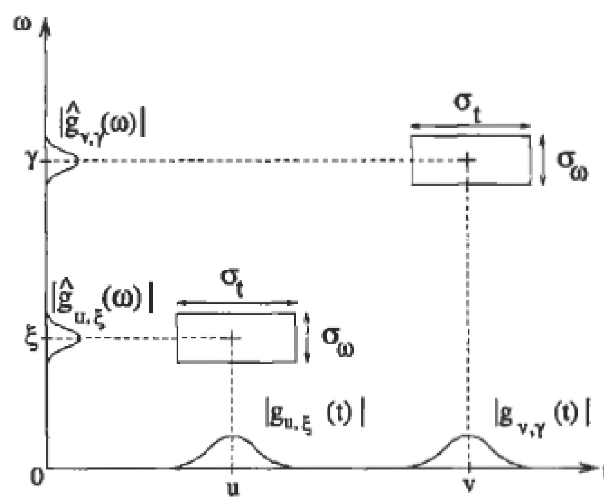
W FT

$$Sf(u,\xi) = \int_{-\infty}^{+\infty} f(t)\, g^*_{u,\xi}(t)\, dt = \int_{-\infty}^{+\infty} f(t)\, g(t-u)\, e^{-i\xi t}\, dt$$

$$Sf(u,\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\omega)\, \hat{g}^*_{u,\xi}(\omega)\, d\omega.$$

Time invariant filter

$$Lf(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(\omega)\, \hat{h}(\omega)\, e^{i\omega t}\, d\omega.$$



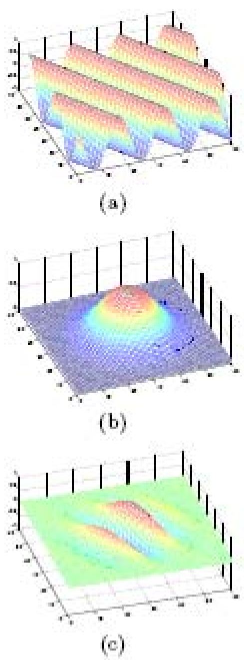
Gabor filters

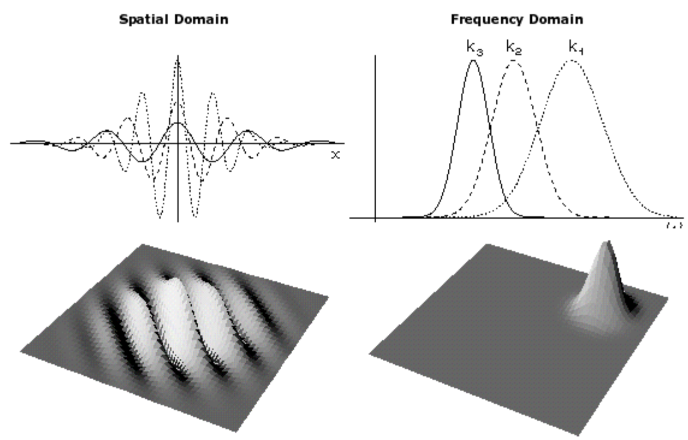
g =Gaussian modulated with an oriented sin wave

$t=(x,y)$

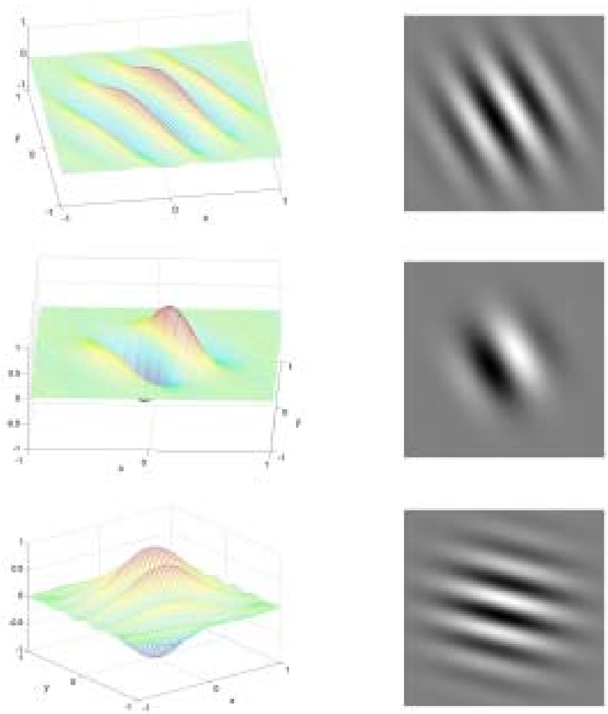
$$g_{\tau\xi\Phi}(x,y)=\frac{1}{2\pi\tau^2}e^{-\frac{x^2+y^2}{2\tau^2}}e^{2\pi i\xi(x\cos\Phi+y\sin\Phi)}$$

Gabor atom: $g_{u,\xi}(t)=g(t-u)e^{i\xi t}.$



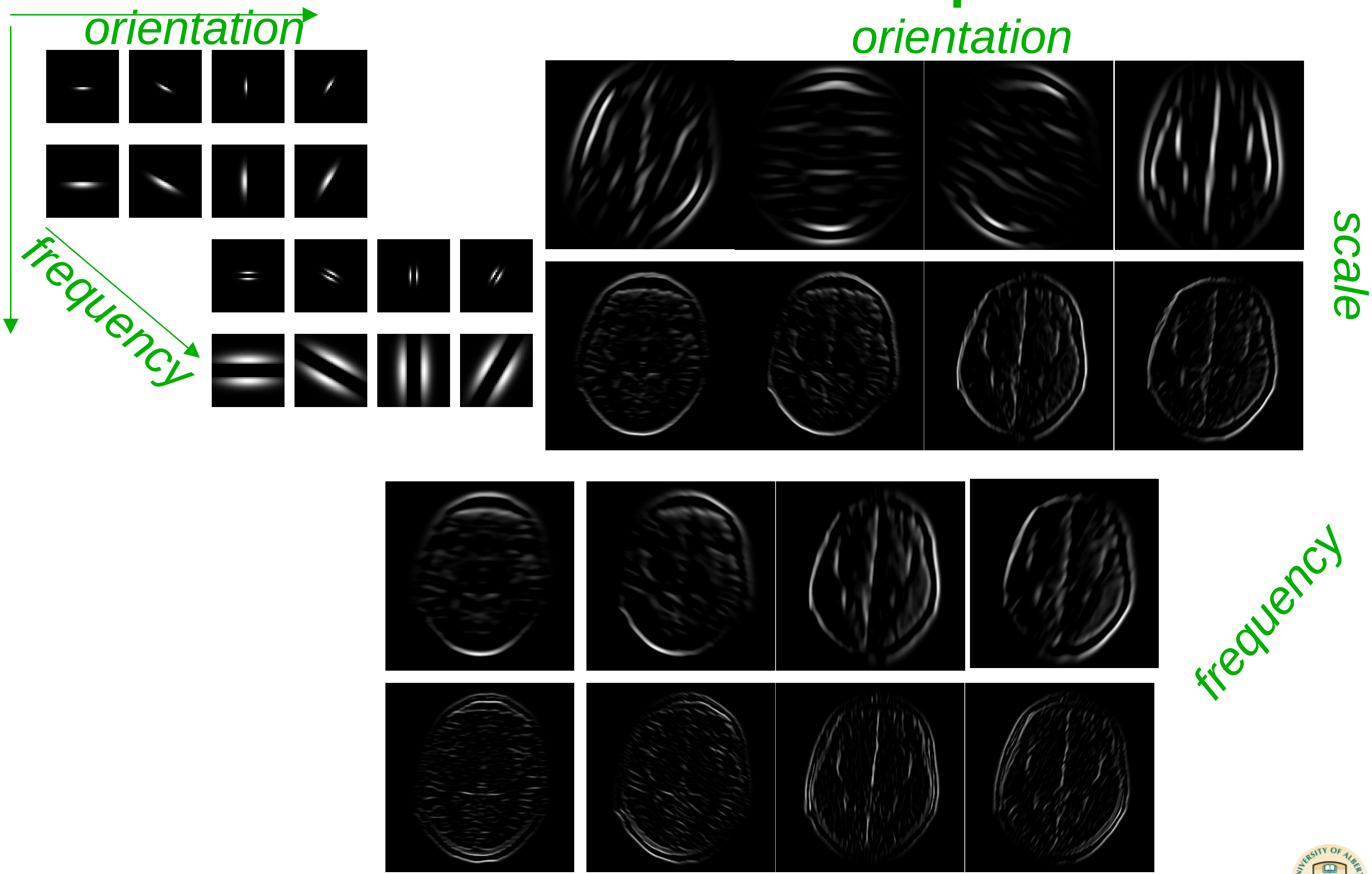


Gabor filter formation



Examples of Gabor filters

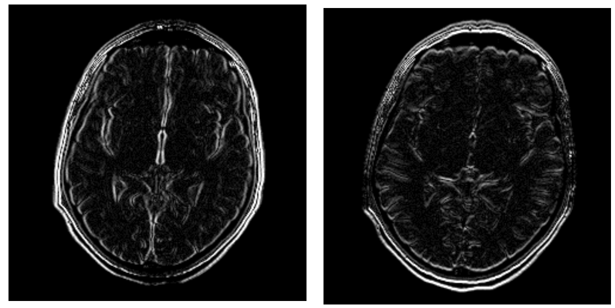
Gabor filters examples



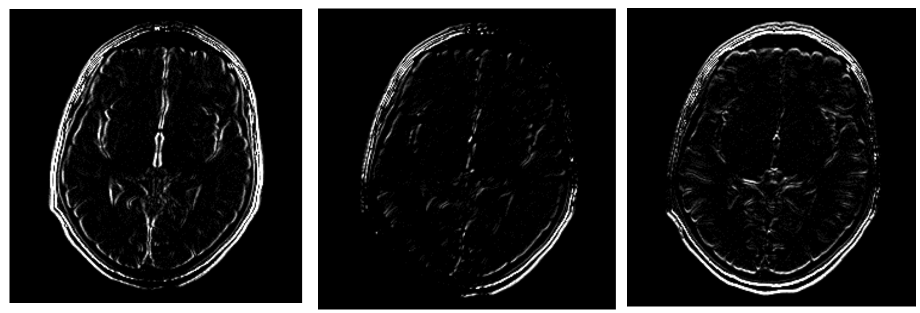
Structure tensor

Image gradient

∇I



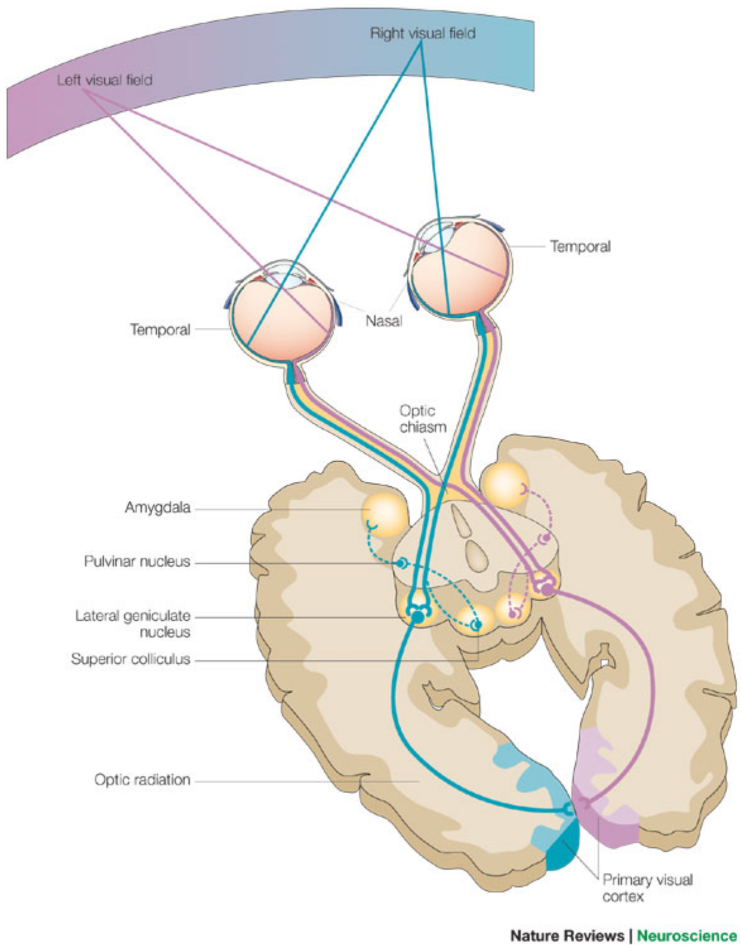
Structure tensor



Smoothed tensor $G_\tau * (\nabla I \nabla I^T) = \begin{pmatrix} G_\tau * I_x^2 & G_\tau * I_x I_y \\ G_\tau * I_y I_x & G_\tau * I_y^2 \end{pmatrix}$

- *Semi-definite matrix*
- *dominant orietation = corresponding to biggest eigenvalue*
- *magnitude – trace*
- *homogeneity of orientation = smallest / biggest eigenvalue*
- *no scale ! – based on linear diffusion (Total Variation flow)*

Motivation for use of Gabor filters



- Simple cells in the primary visual cortex have receptive fields (RFs) which are restricted to small regions of space and highly structured [Marcelja 1980, Jones & Palmer 1987].
- Recent examinations, among others the one by [Jones & Palmer 1987] showed that the response behavior of simple cells of cats corresponds to local measurements of frequencies.

resample Gabor filters

Statistics on natural images

Statistics of filter responses on natural images

- Scale invariance

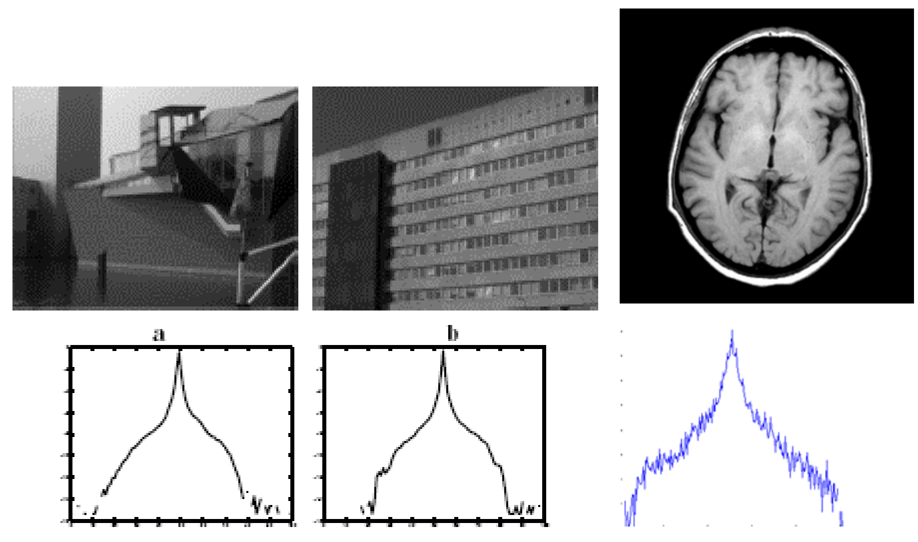
  marginal distribution of stats remain unchanged

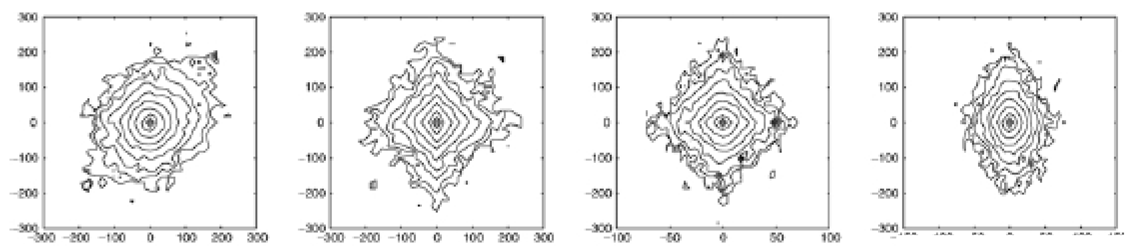
- Non-Gaussian marginal statistics

  large correlations across scale

- Non-Gaussian joint statistics

  dependencies across scale, orientation, position





Different spatial offsets | Different orientation | Differer scale

Models for images

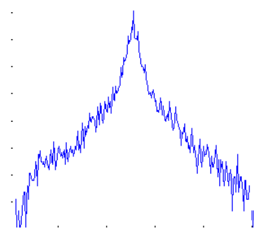
1. Statistical models in image space

- MRF

$$p(\mathbf{I}) = \frac{1}{Z}\exp\left\{ \sum_{\vec{v}} g(\mathbf{I}(\vec{v})) + \sum_{\vec{u},\vec{v}} \beta_{\vec{u}-\vec{v}} \mathbf{I}(\vec{u})\mathbf{I}(\vec{v}) \right\},$$

$$p(\mathbf{I}) = \frac{1}{(2\pi\sigma^2)^{n/2}} |B|^{1/2} \times \exp\left\{ -\frac{1}{2\sigma^2}(\mathbf{I}-\mu)^T B (\mathbf{I}-\mu) \right\},$$

- Analytical densities of filter responses
  - Generalized Laplacian
  - Student-t distribution
  - […]



Models for images

2. Image manifolds : define a lower dimensional manifold in the space of NxM matrices

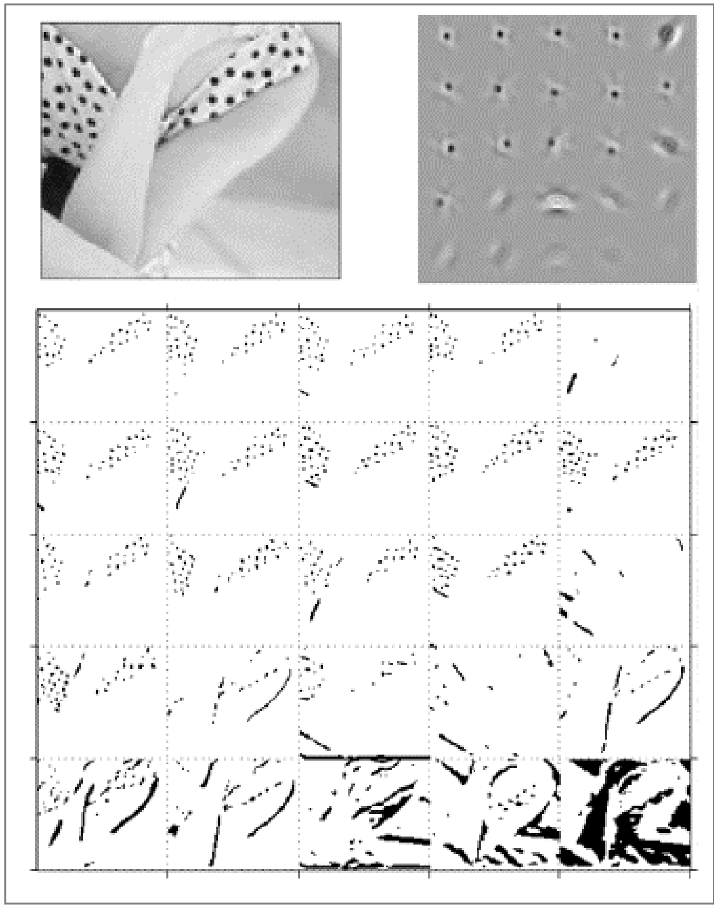
- Linear local subspaces

  PCA – stats. of projected coef.

  ICA - minimize correlation

  Local linear embedding

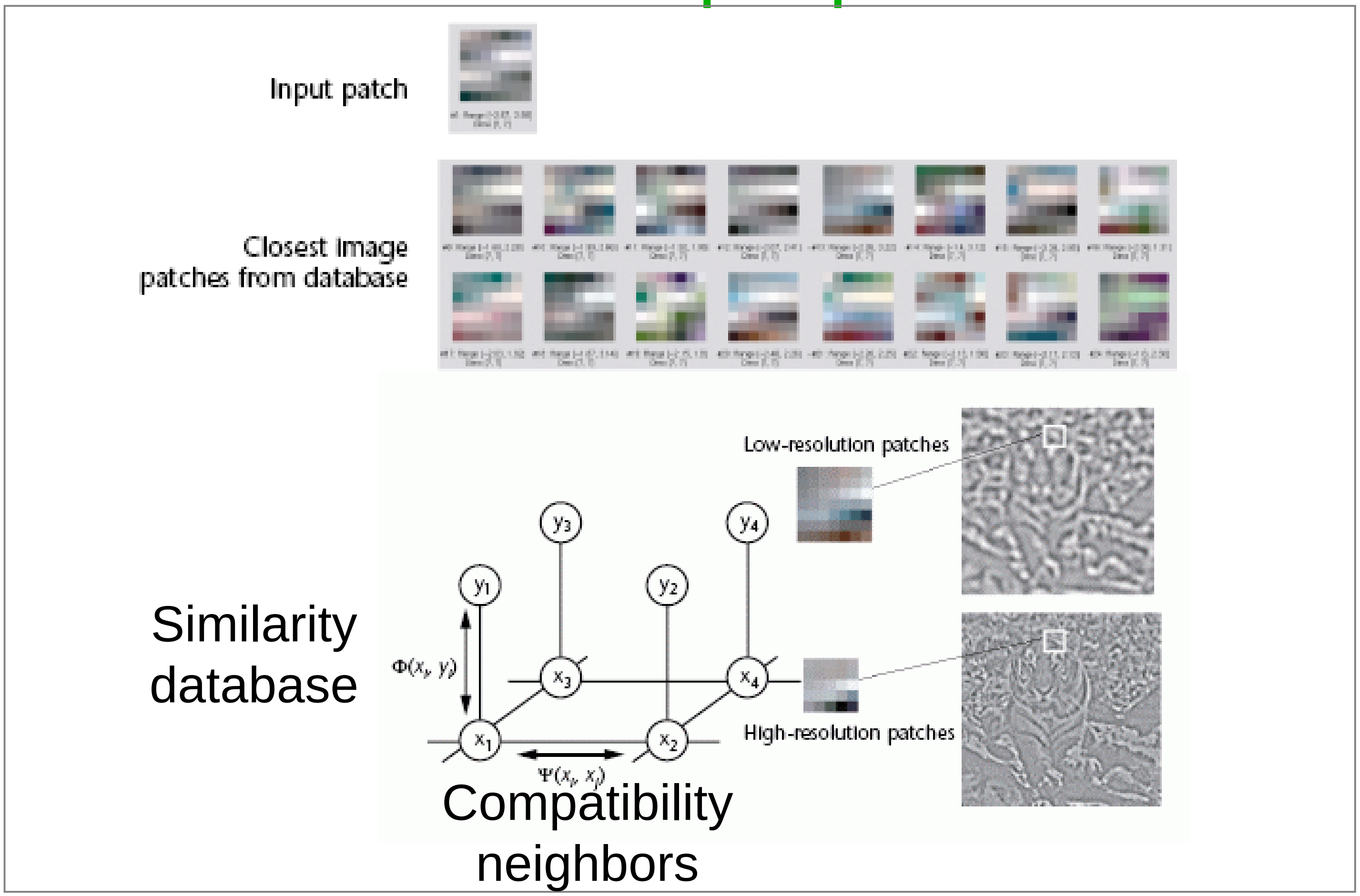
  Texons – higher local structures



[Texons- Malick IJCV 2001]

Models for images

3. Database of example pathes :



[Super-resolution – Freeman et al. 2002]