CMPUT 414/498: Winter 2006

Written Assignment 1, due date: Feb 14th 2006, Max. marks: 50
Email to taowang@cs.ualberta.ca or hand in class on or before Feb 2nd, 2006

1. 3D Transformations

a) What is the homogenous transformation matrix for:

a) Rotate of 30 degrees around x-axis
b) Followed by Translation of (10, 20, -30)
c) Followed by Rotation of 45 degrees around y-axis
d) Followed by Rotation of 60 degrees around z-axis

b) There are two point sets {(X1, Y1, Z1)} and {(X2, Y2, Z2)} as follows:

X1	Y1	Z1	X2	Y2	Z2
67	130	50	63	130	53
72	114	16	69	114	19
72	134	19	69	134	22
74	110	30	71	110	33
74	115	16	71	115	19
79	117	26	76	117	29
80	112	23	77	112	26
80	117	26	77	117	29
81	116	69	77	116	72
83	118	67	79	118	70
86	130	48	82	130	51
93	114	16	90	115	19
94	111	16	91	112	19
94	112	75	90	112	78
96	110	15	93	111	18
96	111	76	92	111	79
97	109	77	93	109	80
100	115	15	97	116	18
101	112	15	98	113	18
102	112	15	99	113	18
104	111	85	100	111	88

We know that {(X2, Y2, Z2)} is transformed from {(X1, Y1, Z1)} with certain transformations. Each row in the table is the coordinates of two corresponding points. Please calculate the transformation matrix. (Hint: you may use MATLAB or some other mathematical tools, or write your own codes to solve this question. You do not have to submit your codes, but you must report your ideas. Only the transformation matrix is NOT enough.)

c) What is the homogeneous matrix that converts the vector from the origin to (1, 1, 1) to the vector from the origin to (2, $1/\sqrt{2}$, $1/\sqrt{2}$)?

2. Perspective projection
Does perspective projection map straight lines into straight lines? Prove your answer.

3. Huffman coding
Consider an 8x8 image with eight possible gray levels, 0,1,2,3,4,5,6,7 as shown below:

0 0 0 0 0 0 0 0
0 0 0 0 1 1 1 1
2 2 2 2 2 2 2 2
3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3
4 4 4 4 5 5 5 5
6 6 6 6 7 7 7 7

(a) Calculate probabilities of eight gray levels.
(b) Calculate the entropy.
(c) Calculate the average length of fixed-length code and code efficiency. Code efficiency $= \frac{H}{L}$, where H is the entropy of a source and L is the average code length for the coding scheme.
(d) Apply reduction process to construct a compact variable-length Huffman code, illustrate the codeword construction process, and calculate code efficiency with respect to entropy.
(e) What is the minimum size of the compressed image taking into account the code table. Justify that your design is of the code table, and why it is efficient.

4. DPCM coding
(a) According to the correlation of adjacent pixels, how would you order the above 8x8 image as a 1D series, calculate its standard deviation.
(b) By minimizing the mean-squared prediction error, derive the first and second order predictor for the 1D series.
(c) Tabulate the differential image of the first order predictor and calculate its standard deviation and the size of its Huffman encoding.
(d) What is the total size of the compressed image considering the code table and minimal header information?

5. Given the same stereo set up as used in the experiments in the "Optimal discretization for stereo reconstruction" paper on the course web page, what will be the best distribution of pixels for reconstruction in the range of 40 to 80 cm? Draw a picture to demonstrate the best distribution

(like Figure 3 – Figure 6 of the paper).

6. Consider a stereo set up with the following parameters: Baseline (dx) = 100 mm; f = 10 mm; resolution of CCDs (approximately 1 cm x 1 cm size) is 101 x 101 pixels (0 … 100, 0 … 100); optical axes of both cameras passing through pixel (5, 5).

Given a 3D point at (23 mm, 33 mm, 1180 mm)

a) What is (x_l, y_l)?
b) What is (x_r, y_r)?
c) What is the estimated 3D point based on stereo reconstruction?