# Monte Carlo Tree Search in Go

Adrien Couëtoux and Martin Müller and Olivier Teytaud

**Abstract** Monte Carlo Tree Search (MCTS) was born in Computer Go, i.e. in the application of artificial intelligence to the game of Go. Since its creation, in 2006, many improvements have been published. Programs are still by far weaker than the best human players, yet the gap was very significantly reduced. MCTS is now widely applied in games, in particular when no trustable evaluation function is readily available. This chapter reviews the development of current strong Go programs and the most important extensions to MCTS that are used in this game.

## 1 Introduction

This chapter describes the application of Monte Carlo Tree Search (MCTS) methods to the game of Go. Computer Go was the first major success story for MCTS, and many important techniques were developed in this context. Go remains the best studied application, with a huge amount of practical work as well as many publications over the last seven years.

The game of Go is a perfect information turn-based two player board game. As usual in board games, actions are called moves. In Go, a normal move consists in placing a single stone on the Go board, plus removing any captured opponent stones from the board. Special move types are passing and resigning.

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The state of a Go game is described by the location of black and white stones on the board, plus the move history, including who plays next. History is needed to check the Ko rules which forbid certain types of repetition (see Section 1.2). Some versions of the rules also require keeping track of the number of captured stones.

Notation: Throughout the chapter, moves are denoted by $m$, states by $s$, and the player to play in state $s$ by $player(s)$.

## 1.1 The importance of Go

The game of Go originated in China more than 2000 years ago. It has simple rules, but advanced tactics and a striking strategic level. It is the oldest known abstract board game, and is actively played by millions of people. There is a large community of hundreds of professional Go players, mainly in Asia. These players study, teach and play Go full-time. Compared to many other classical games, progress in Computer Go has been much slower, and it has been considered a “grand challenge” of artificial intelligence. The recent breakthrough of MCTS has, for the first time, led to strong computer programs, and has even opened up perspectives to approach the level of the best human professionals. This level has already been achieved on small boards of size $9\times 9$ and smaller.

A key feature about Go is its great strategic *depth*. The depth of a game [53] is defined as the maximum number of players $p_1,\dots,p_{depth}$ such that, if $1\leq j< i\leq depth$, player $p_j$ wins with probability at least 60% against $p_i$ . Depth is not always clearly defined, and it relies on a set of players, but it makes it possible to compare different games. Go has a depth around 40, which is much more than any other known game.

## 1.2 The rules of Go

Go is played by placing stones on the intersections of a square board. The standard board size is $19\times 19$, but smaller sizes are popular for teaching and quick games. Black plays first, then the two players (black and white) alternate until the game ends with two or three consecutive passes, or with a resignation. Stones never move on the board, but are removed when captured. Examples and details of these rules are given in Fig. 1. The Ko rule prevents players from creating a potentially endless loop by repeating a previous situation. This rule is explained in Fig. 2. At the end, the player with the larger score is the winner. The exact scoring method depends on the rules used. The simplest method used by most Go programs is a variant of Chinese rules, which counts both stones and surrounded empty points, called territories, for each player (see Fig. 3). The score can be adjusted by a fixed value called Komi. This is used to compensate for the advantage of the first player, or sometimes to compensate for the different strength of the players. The more usual method to

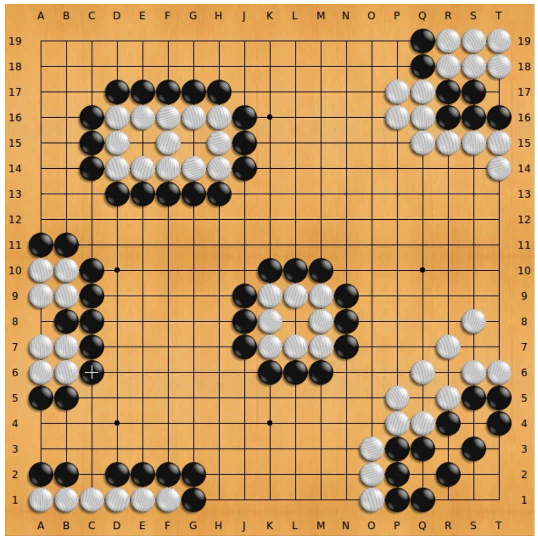



**Fig. 1** Rules of the game of Go.
**Blocks:** A block of stones is a maximal 4-connected set of stones of the same color. For example, {J7,J8,J9} is a block, but {J7,J8} is not (not maximal) and {J7,J8,J9,K6} is also not (not 4-connected). A block of stones can be referred to by any of its stones: block J7 is the same as the block J8.
**Liberties:** The empty intersections adjacent to a block are called its liberties. For example, block A1 has a single liberty at C2, while block A2 has three liberties at A3, B3 and C2.
**Capture:** A block of stones is captured if the opponent fills its last liberty. For example, Black C2 captures the White block {A1,B1,C1,D1,E1,F1}. Captured stones are removed and these intersections become empty again. It is possible to capture several blocks in one move. For example, Black C8 captures the two white blocks A6 and A9.
**Suicide:** In most variations of the Go rules, it is illegal to play a stone that fills the last liberty of the player's own block, while not capturing any opponent. For example, White L8 is suicide whereas Black L8 is a capture and is allowed. White A8 is also suicide. White can play either one of E15 or G15 but not both. Black can not play either move. T17 is the last liberty of both the white block T18, and the black block T16. Whoever plays first can capture the other.
**Chains, groups etc:** Closely related or virtually connected sets of blocks are often called chains or groups. The notation is not consistent in the literature. Typically, a group is treated as a unit and all its blocks are expected to live together or die together. Cutting off a group from other blocks is considered an important event in a game.

compensate strength differences is to give handicap stones: Black is allowed to make several consecutive moves at the start of the game.

For more information about the game of Go in general, see [8, 68]. For information about rules and variations, see [69].

Two important Go concepts are eyes, and the safety of blocks of stones. Group E16 in Fig. 1 has two eyes at E15 and G15. Playing either of these moves would be suicide for Black (cf Fig. 1). Therefore, a group with two eyes can not be killed by

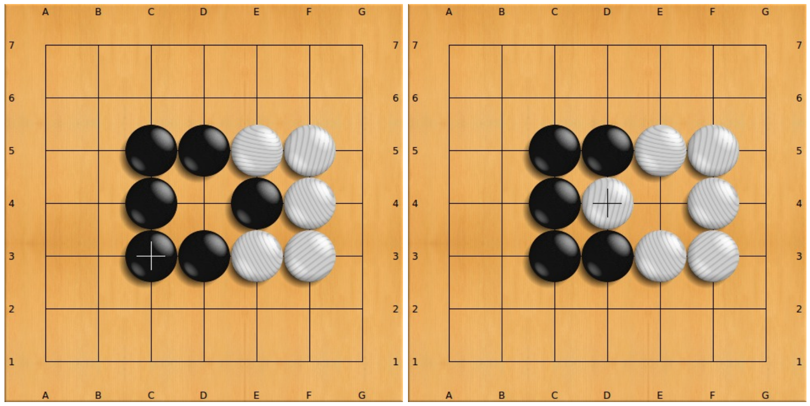



**Fig. 2 The Ko rule:** In the figure on the left-hand side, white can capture a single stone with D4, leading to the position on the right. The Ko rule forbids Black from immediately recapturing on E4 which would repeat the position on the left. All Ko rules prevent such simple cycles of length 2. Rules differ in their handling of longer, more complex cycles [69].

the opponent; it is safe unless its owner fills its own eyes. In contrast, block K7 has only one eye; it can be killed by Black L8.

**Computational Complexity.**

A variant of the rules, namely the Japanese rules, is known to be Exptime-hard [54]; more precisely, this (not fully defined) set of rules contains a well defined set of positions which is Exptime-complete. The Chinese variant is completely defined but its complexity is unknown; it is in the (huge) gap between Pspace-hard [22] and Expspace.

## *1.3 Computer Go before Monte Carlo Tree Search*

Work on Computer Go started fifty years ago. Early programs were rule-based systems, and dealt with basic tactics such as ladders. The advent of affordable personal computers in the mid 1980's led to a big boost in popularity, and to the creation of the first regular tournaments such as the Ing cup.

Wilcox describes the early development of Computer Go up to the late 1970's [70]. The first large-scale Go program was Interim.2, written in Lisp by Reitman and Wilcox [51]. Reviews of the development of Computer Go up to about the year 2000 and detailed survey papers covering "classical" Computer Go were published about ten years ago [6, 45, 46].

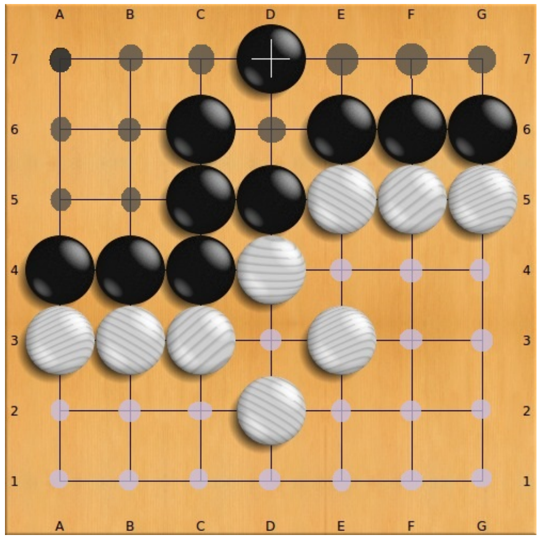



**Fig. 3 Scoring:** When no player is willing to keep playing (both play a pass) the game is stopped and we count who has won. E.g. for Chinese rules, the points for Black are (i) the black stones on the board (ii) the areas enclosed by black stones. The points for White are (i) the white stones on the board (ii) the areas enclosed by white stones (iii) a bonus for White, termed Komi, e.g. 5.5, aimed at compensating Black's advantage of first play. Here, on a $7 \times 7$ board, Black has 21, whereas White has $28 + 5.5 = 33.5$; White has won by 12.5 points. Players can play until they see that whatever additional stone they play, it will be captured; at some point there will be no more legal move. The game is usually stopped far before there is no legal move.

One measure of progress were the yearly handicap exhibition matches between the Ing Cup winner and three young human players, often of *insei* (student professional) strength. The strength of the computer program would then be measured by how many stones it needed to win (the less stones, the better the program). The first hurdle at 17 stones handicap was taken in 1991 by Mark Boon's program GOLIATH. In 1997, Chen Zhixing's HANDTALK won on 11 stones. At the time of the last Ing Cup in 2000, programs had not yet won a game at 9 stones, which is considered the highest handicap in regular use. Nick Wedd maintains a website with detailed information, including game records, of all man-machine Go matches [67].

Progress was slow. As computers got faster, programs started to use more specialized tactical searches such as block capture and life and death. GOLIATH implemented a successful goal-oriented global search. Other programs such as HANDTALK, THE MANY FACES OF GO and GO++ also started using global search. The main obstacle for progress with using traditional alpha-beta based methods in Go was the lack of a fast but reasonable evaluation function.

One notable development of the years leading up to the Monte Carlo revolution was the rise of GNU GO, the first strong open source Go program [28].

Typical ingredients of a "classical", pre-Monte Carlo Go program were:

- A fast implementation of a Go board, including rules, executing and taking back moves, blocks and their liberties
- A ladder reading module, sometimes implemented on a specialized board with reduced “lazy” updates
- An *influence function* which radiates out from stones, and allows the program to assign a “likely owner” to each empty point
- A *pattern matching system*, typically containing several thousand hand-made patterns, with extra attributes such as liberty counts
- A sophisticated set of data structures for higher-level units on a Go board, such as chains, groups, and territories
- A set of heuristics for recognizing and evaluating eyes and potential eyes
- Specialized selective search modules for capturing blocks of stones, and for life and death of groups
- A highly selective, shallow global search
- An opening book containing standard *joseki* corner sequences

Since the overall problem of playing Go well appeared intractable, quite a bit of work has gone into specialized approaches to handle specific subproblems of Go. Representative examples are life and death solvers [71, 39], specialized searches for capturing blocks of stones [63], solving endgame puzzles using methods from combinatorial game theory [44], proving the safety of stones and territories [4, 43, 49, 50], and solving seki [48].

Van der Werf has solved Go for several small but highly nontrivial board sizes including $5 \times 5$ [64], $5 \times 6$ and $4 \times 7$ [65]. An overview table is shown on `http://erikvanderwerf.tengen.nl/mxngo.html`.

### Towards MCTS for Go

While the top-level structure of MCTS-based Go programs is completely different from classical ones, some components have been kept, reintroduced, or further refined:

- Fast incremental data structures for maintaining a board, blocks and liberties
- Tools for dealing with large numbers of game records from master players or from computer players; tools for automated testing and parameter tuning using self-play and tournaments
- Zobrist hashing [74]: the Zobrist hash is a fast and fair method for mapping from Go boards to integers. It is used for storing positions in transposition tables. This can save memory and computational cost by avoiding duplicates in the tree. However, dealing with the resulting cyclic and acyclic graphs in MCTS has its own challenges [16, 55].
- The AMAF and RAVE methods described in Section 3.1 are related to, but different from the *history* heuristic [56] used in alphabeta search.
- Heuristic Go knowledge in the form of pattern matching (Section 3.2), expert rules (Section 3.3), and opening books (Section 3.4).

There are also approaches based on reinforcement learning. NeuroGo [25] uses reinforcement learning (Section 1.5) with neural networks, and RLGO [60] uses both offline and online learning of weights for patterns up to size $3 \times 3$.

Bernd Bruegmann designed the first Go evaluation function using Monte Carlo simulations and implemented it in his *Gobble* program [10]. His approach simulates many random games from a given state, and estimates the move values by their winning probability. This algorithm missed the crucial second component of MCTS, the tree search, and its performance was much weaker than the classical programs at the time. Therefore, there was little follow-up work for several years.

## 1.4 The Bandit Literature

In this section, we introduce the bandit literature, which is a theoretical, maybe more than practical, ground for MCTS. Developed during the recent decades, this body of work is devoted to the problem of choosing one action $a$ in a set of $K$ *options* (also termed *arms*), at each time step $t \in \{1,\dots,T\}$. One obtains a stochastic reward each time an action is selected. Some of the common objectives are to maximize the expected total reward from 1 to $T$, or to maximize the expected reward obtained at the last time step. In the former case, one needs to balance ***exploration*** and ***exploitation***. Focusing on exploration only would mean to choose actions uniformly in order to explore as many options as possible, to avoid missing a good one. On the other hand, focusing on exploitation would mean choosing the action that has the best empirical average reward. This balance is at the heart of successful approaches to multi-armed bandit problems.

In the case of Monte Carlo Tree Search, bandit algorithms are used for choosing which moves should be simulated more often in a given state (Section 2). The goal is to balance the exploration of actions that we know little about versus the exploitation of actions that returned the highest rewards in the past (the incentive being that we may want to simulate these promising moves to confirm their high value).

The bandit problem [42, 1] is formally described as follows:

- There are time steps $1,2,\dots,T$.
- There are arms, also termed options, $1,2,\dots,K$.
- At each time step $t \in \{1,\dots,T\}$,
  - An arm $a_t \in \{1,\dots,K\}$ is chosen. It is indeed chosen by a bandit policy $\pi$, with $a_t = \pi(a_1,\dots,a_{t-1},r_1,\dots,r_{t-1})$.
  - Then, a reward $r_t = r_{a_t,t}$ is obtained. The random variables $r_{a,t}$ for $a \in \{1,\dots,K\}$ and $t \in \{1,\dots,T+1\}$ determine the problem.
- Other quantities of interest are
  - $n_m(a) = Card\ \{t \in \{1,\dots,m\}; a_t = a\}$, the number of time steps $\leq m$ at which arm $a$ has been chosen.
  - $r_m(a) = \sum_{t\in\{1,\dots,m\};a_t=a} r_t$, the total reward of arm $a$.

– $\hat{q}_m(a) = r_m(a)/n_m(a)$ average reward of arm $a$ (defined if $n_m(a) > 0$; set to $+\infty$ otherwise).
– the Upper Confidence Bound (UCB) score:

$$score_{ucb,t}(a) = \underbrace{\hat{q}_t(a)}_{exploitation} + \underbrace{C\sqrt{\log(t)/n_t(a)}}_{exploration} \tag{1}$$

and its associated policies $\pi_{ucb}$ which chooses at time $t$ the arm $a$ which maximizes $score_{ucb,t-1}(a)$ (randomly breaking ties). The ***exploration*** part represents the incentive to simulate arms that we have little knowledge about. The ***exploitation*** part represents the incentive to simulate arms that have shown good results in the past simulations.

- Given $a_1,\ldots,a_T$ and $r_1,\ldots,r_T$, a recommendation policy $R$ decides $a = R(a_1,\ldots,a_T,r_1,\ldots,r_T) \in \{1,\ldots,K\}$. Classical recommendation policies include

$$\begin{aligned} R_{EBA} &= \arg\max_a \hat{q}_T(a) \\ R_{MPA} &= \arg\max_a n_T(a) \\ R_{LCB} &= \arg\max \hat{q}_T(a) - C\sqrt{\log(T)/n_T(a)} \end{aligned}$$

where EBA, MPA, LCB stand for Empirically Best Arm, Most-Played Arm, Lower Confidence Bound respectively.

In the case of MCTS,

- the policy $\pi$ is used for choosing which move is simulated from the current situation;
- the recommendation policy $R$ chooses which move is actually played.

The *expected cumulative regret* (ECR) is

$$ECR = \max_{b\in\{1,2,\ldots,K\}} \sum_{t=1}^{T} \mathbb{E}(f(b) - f(a_t)).$$

The *expected simple regret* associated to a bandit policy $\pi$ and a recommendation policy $R$ is $\mathbb{E}(r_{T+1,a})$ with $a$ chosen by $R$ and $a_1,\ldots,a_T$ decided by $\pi$.

Bandits can be considered in a stationary or a non-stationary setting:

- The stationary case, corresponds to the case in which the $(r_{t,a})_{t\in\{1,\ldots,T+1\}}$ are independently identically distributed according to some unknown distribution $d_a$, usually assumed to lie in some interval. The stationary case can be handled by different algorithms; depending on the exact criteria, the optimal policy can be Upper Confidence Bound ($\pi_{ucb}$) with recommendation $R_{MPA}$, or the uniform policy $\pi_{uniform}$ with recommendation $R_{EBA}$.
- The non-stationary case is harder to define, as many distinct non-stationary problems can be defined. Bandits involved in MCTS are non-stationary, with some special properties (convergence, for $t \to \infty$, of $\hat{r}(a,t)$ either to 0 or to 1). The

bandit part in MCTS is indeed not fundamental in the success of a MCTS for a large problem; the playout policy (Section 3.2.3) and the bias in the tree (Section 3.2.2), both in particular using domain knowledge, are much more important. But a MCTS programmer should keep in mind the principle of exploration/exploitation. Discounted variants were proposed in [41]; they are successful in some bandit problems but have not been successfully tested in MCTS problems.

Adversarial bandits are a related family of problems in which $r_{t,a} = M(a_t, b_t)$ where $M$ specifies the problem, $b_t$ is chosen by an opponent aware of $a_1, \ldots, a_{t-1}$ and $r_1, \ldots, r_{t-1}$ but not aware of $a_t$. Such bandits are solved optimally in [33, 2] and applied to MCTS for games with simultaneous moves in [27].

## *1.5 Reinforcement learning background*

Reinforcement learning [62] is the setting where an *agent* (also sometimes called *controller*) interacts with an *environment* in order to learn how to choose an *action* (or a *move* in the case of Go) depending on the current *state* of the system. At initialization, the agent has no knowledge of what good or bad actions look like. He needs to interact with the environment to observe the consequences of different actions, and learn what are the best actions. The signal given by the environment is often a numerical value called *reward*, that the agent tries to maximize, over a certain period of time. In the case of Go, the reward can be 1 when a game is won and 0 or $-1$ when a game is lost. In the context of a mimicking approach, the reward can be 1 when the agent reproduces the moves by an expert and 0 otherwise (see e.g. Section 3.2.1 for such a case in the game of Go). The time horizon can be finite, like in the game of Go, or infinite. In the latter case, or with large time horizons, one often needs to discount the future rewards, in order to have finite objective function values. In simulation balancing (Section 3.2.3) the reward is the similarity between the percentage of wins for black in a given situation and the probability as estimated by an expert. Reinforcement learning problems can be model based or model free. In the former case, one assumes that a model of the environment is available, and that for any couple state-action, one can simulate a transition, i.e. how the environment would react to such couple. In the latter case, no such assumption is made, and one usually only has historical data of observed transitions (i.e. tuples containing a state, an action, a resulting state and observed reward). For example, mimicking in Go is based on databases of games.

The most classical algorithms in reinforcement learning are either *actor* methods (based on learning directly a function which maps inputs to outputs), or *critic* methods (learning an evaluation function, i.e. a function which, for maps situations to probabilities of winning). In the case of Go, [18] learns a function as in actor methods, but the crucial new component is the Tree Search on top of it. [57] (using neural networks) and [60] (using a linear representation by all $3 \times 3$ patterns) are of the critic type.

## 2 Basic Monte Carlo Tree Search

The idea is to use Monte Carlo simulations to evaluate the value of all available moves, in order to select the best one. In the particular case of a 2 player game like Go, uniform sampling based simulations are not enough to provide an accurate estimation of the moves' values. Ideally, one would want to simulate all possible sequences of moves, and weight each opponent's move with the probability that he would choose that specific move. But this would require an almost infinite computational budget (due to the number of possible sequences), and a perfect knowledge of the opponent's move preferences. Both of these being out of reach, the key idea of Monte Carlo Tree Search (MCTS), as defined in [18], is to bias the sampling of moves, to invest most of the computation time in the actions that look the most promising, using bandit algorithms (see Section 1.4). This focus on good moves is how MCTS deals with intractably large search space. The other challenge is the unknown opponent's strategy. To deal with it, the bias of move sampling in MCTS is designed so that, asymptotically, the best move will be infinitely more simulated than the other moves. The ultimate goal is to approximate the minimax tree derived from the current state. Combining this goal with the biasing of simulations means that MCTS will only approximate the minimax tree for the subtree corresponding to the most promising moves.

Practically speaking, given a state $s$, MCTS uses the computation time to build a tree that represents a subset of all possible trajectories from that state $s$. The nodes of the tree correspond to reachable states, and the arcs represent available moves. The longer MCTS runs, the larger the tree grows, and the more information is available. This information is updated after each iteration, and used to bias the following simulations. When time runs out, one uses that information to select the move that is the best according to a chosen criterion (e.g. the most simulated move). We present the MCTS algorithm in detail below.

### 2.1 Useful notations

Simulations will be denoted by $g_1, \ldots, g_t$, i.e. $g_t$ is the $t^{th}$ simulation performed during the thinking time. Collected statistics are the following:

- $\hat{q}_t(s,m)$ denotes the frequency at which, in simulations $g_1, \ldots, g_t$, simulations including move $m$ in state $s$ have been a win for the player to play in state $s$.
- $\hat{V}_t(s)$ is the frequency at which, in simulations $g_1, \ldots, g_t$, simulations including state $s$ have been a win for the player to play in state $s$.
- $n_t(s,m)$ is the number of simulations, in $g_1, \ldots, g_t$, which include move $m$ played in state $s$.
- $n_t(s)$ is the number of simulations, in $g_1, \ldots, g_t$, which include state $s$.
- $n_{rave,t}(s)$ is the number of simulations, in $g_1, \ldots, g_t$, which include move $m$ and state $s$, with $m$ played by $player(s)$ *in* state $s$ or *later* than state $s$. RAVE stands

for "rapid action value estimates", to be detailed later. RAVE values, originally known as All-Moves-As-First (AMAF), were described in [10], in the context of Monte Carlo simulations without tree search; they were adapted to the MCTS context in [30].

- $\hat{q}_{rave,t}(s,m)$ is the frequency at which simulations, in $g_1,\ldots,g_t$, have been a win for $player(s)$ and contain move $m$ played by $player(s)$ *in* state $s$ or *later* than in state $s$.
- $\mathscr{T}$ the tree currently stored in memory by MCTS
- $\mathscr{M}(s)$ the moves available in state $s$

As far as possible, we remove the subscript $t$, but we should keep in mind that all these quantities evolve dynamically, depending on the results of simulations. After each simulations, these quantities are updated.

## *2.2 Main loop of MCTS*

We explain here more formally how MCTS works. The algorithm can be described as follows: given a state $s_0$, it creates a tree $\mathscr{T}$ where it stores the information gathered during simulations of the game (it is thus initialized with a single node, its root, the state $s_0$). MCTS then repeats the main loop, here called GrowTree, until the computation time is over. It then uses the information stored in $\mathscr{T}$ to select a move, implementing a function that we call BestAction($\mathscr{T}$). BestAction is also termed the *recommendation* policy, or final selection function.

A commonly used final selection function is to pick the most simulated move. But there are other options available, inspired by the bandit literature (see Section 1.4), such as EBA or LCB.

The GrowTree function, the main loop of MCTS, contains the core features of MCTS, and will be the focus of this section. It can be divided in four steps [12], as illustrated by Figure 4, borrowed from [9]:

1. *Selection* (tree part): starting in the root node (the current state $s_0$), and a selection policy, one travels down the tree, until one decides to expand the tree by simulating an unvisited node;
2. *Expansion*: given a node $s$ in the tree, one chooses a feasible move $m$ that has not been visited yet, and adds the resulting state $s'$ in the tree;
3. *Simulation*, also termed *playout*: from that new node $s'$, one simulates a possible trajectory until a final state is reached, where the result of the game is known;
4. *Backpropagation:* given the last simulation's result, the tree is updated, typically by updating the statistics (average win ratio, number of simulations, etc as explained in Section 2.1) in all the nodes visited during this last simulation.

We formally describe the MCTS algorithm in Alg. 1. We generally denote the current tree stored in memory $\mathscr{T}$. It is initialised as a single node, its root, the initial state $s_0$.

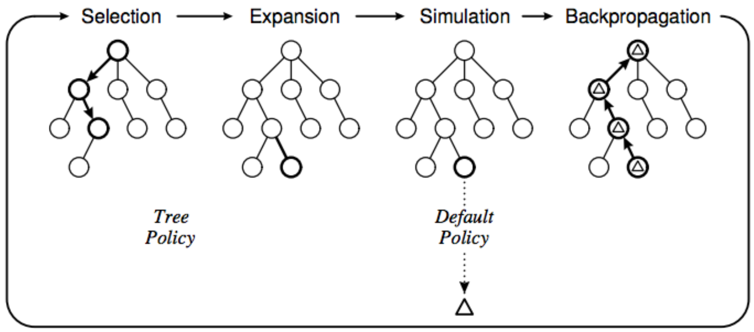



**Fig. 4** The four steps of the main loop of MCTS. Each iteration of the loop corresponds to one possible sequence of states and moves, and adds information to the tree.

**Algorithm 1** MCTS algorithm

**Require:** initial state $s_0$, time budget $B$
**Ensure:** a chosen move $\tilde{m}^*$
Initialize $t_0 \leftarrow t$, and $\mathscr{T} \leftarrow s_0$
**while** $t < t_0 + B$ **do**
GrowTree($\mathscr{T}$)
**end while**
**Return:** BestAction($\mathscr{T}$)

GrowTree is the main loop of MCTS. It is iterated as long as the time budget allows it. We formally describe this loop in Alg. 2. The transition function is specific to the problem, and is assumed to be known. In the case of Go, this function models the rules of the game and is deterministic.

This loop clearly shows three essential parts of MCTS: (i) the selection policy, (ii) the simulation procedure, and (iii) the update of the tree. We dedicate the section 2.3 to the selection policy, that is at the core of MCTS, and that was a key factor in the breakthrough for computer Go [40].

As for the simulation procedure, a standard choice is to apply a default policy until a final state is reached, where the result of the game can be computed. An easy choice for that default policy is to choose random moves, but more elaborate default policies can improve the overall performance (see Section 3).

Updating the tree is usually done by storing in each state the number of simulations and the win ratio. Again, the result of one simulation can be used more widely to spread information across the tree.

**Algorithm 2** GrowTree

**Require:** tree $\mathscr{T}$
**Ensure:** adds information to the tree $\mathscr{T}$
  Initialize $s \leftarrow root(\mathscr{T})$
  **while** $s \in \mathscr{T}$ AND !($s$ =final state) **do**
    $m$ =Selection($s$)
    $s$ =Transition($s,m$)
  **end while**
  result=Simulation($s$)
  $\mathscr{T} \leftarrow$ Update($\mathscr{T}, result$)

### 2.3 How simulations are biased inside the tree

$q(s,m)$ is a score used in the algorithm for adapting the simulations. $q(s,m)$ has various definitions in various papers, but a classical version popularized by [40] is as follows:

$$q(s,m) = \hat{q}(s,m) + \sqrt{\log(n(s))/n(s,m)}. \tag{2}$$

This formula is inspired by [42, 1], as explained in Section 1.4 (Eq. 1). We keep notations simple, but special cases are implemented for avoiding $\log(0)$ or divisions by zero. A key point is that the right hand side term should be positive so that exploration is never zero. For instance, one can set $q(s,m) = +\infty$ if $n(s,m) = 0$, in order to explore all moves at least once before actually implementing the above formula. Alternatively, [14] proposes *progressive widening* to slowly increase the number of moves considered from a state $s$ as $n(s)$ increases. Then, when the algorithm reaches a state $s$, it simulates the move $m$ that maximizes the score function $q(s,m)$ (if multiple moves maximize it, pick one randomly). In Algorithm 2, this means:

$$\text{Selection}(s) = \operatorname*{argmax}_{m \in \mathscr{M}(s)} q(s,m) \tag{3}$$

## 3 Popular MCTS Extensions for Computer Go

This section presents the most popular extensions used in MCTS for Go.

### 3.1 Rapid Action-Value Estimates

When Rapid Action Value Estimates, as described in Section 2.1, are used, then Eq. 2 is modified as follows:

$$q(s,m) = \alpha(\hat{q}(s,m) + \sqrt{\log(n(s))/n(s,m)}) \\ + (1-\alpha)(\hat{q}_{rave}(s,m) + \sqrt{\log(n_{rave}(s))/n_{rave}(s,m)}) \quad (4)$$

where $\hat{q}(s,m)$ and $\hat{q}_{rave}(s,m)$ are defined in Section 2.1. This score linearly combines two move scores:

- The classical UCT score consisting of the average payoff $\hat{q}(s,m)$ plus an exploration term; this part is based on simulations in which move $m$ has been played immediately in state $s$.
- The RAVE score of $m$ in $s$ is computed from all simulations in which $m$ has been played in $s$ or *later* in the simulation. These simulations are termed RAVE simulations. Importantly, RAVE simulations are the same as classical simulations; just, the set of simulations used for valuating a move $m$ is (see Section 2.1)

  – $Simulations(m,s)$: Simulations in which $m$ is played in $s$;
  – $Rave-Simulations(m,s)$: Simulations in which $m$ is played in $s$ or later.

  The union of the $Simulations(m,s)$, for all $m$ legal in $s$, is equal to the union of the $Rave-Simulations(m,s)$; but for a fixed $m$, $Rave-Simulations(m,s)$ is much bigger than $Simulations(m,s)$. For a typical state $s$, each simulation containing $s$ appears in only one $Simulation(m,s)$, and in many different $Rave-Simulation(m,s)$.

The RAVE term is, a priori, less precise than the classical UCT term. However, RAVE produces statistics much more quickly. On average, almost half the moves on the board are played at least once by one player over the course of a whole simulation, whereas there is only a single move per simulation from state $s$. RAVE values are therefore most useful in the early phase of a search, when there are only few simulations for a given move. Therefore, the coefficient $\alpha$ must go to 1 when the number of simulations increases, and should be 0 when there are only RAVE simulations in a node. A classical formula [31] is

$$\alpha = \sqrt{n(s,m)/(n(s,m)+3k)}$$

where $k$ is chosen so that when $n(s,m) = k$, the RAVE value $\hat{q}_{rave}(s,m)$ and the classical value $\hat{q}(s,m)$ have nearly the same weight.

In most strong Go programs, RAVE is used and gives a clear improvement. However, it can fail occasionally [47], in the case where a move must be played immediately, but is often bad when played later in a simulation. Figure 5 shows an example where the program FUEGO does not find the only winning move due to its bad RAVE score.

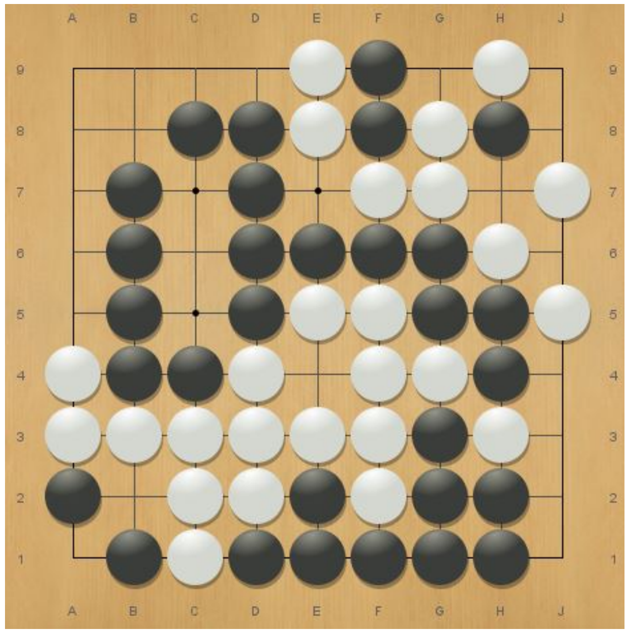



**Fig. 5 Example of misleading RAVE score in FUEGO:** White B2 is the only winning move which avoids a seki. Its RAVE value is very low, since if the liberty at A5 gets filled first, as happens in many simulations, then B2 becomes a very bad self-atari.

## *3.2 Learning and using local Go patterns*

Go players learn many patterns - local shapes of stones - as well as information such as which moves in a pattern are forced, usually good, or bad. All such rules are heuristic and cannot be used for hard pruning.

For example, playing a so-called *empty triangle* is usually bad shape, but in some cases it can be a good move. Figure 6 shows examples. Databases of patterns are used in all strong MCTS programs for both simulations and, to an even larger degree, as in-tree knowledge. While some programs play standard *joseki* sequences instantly, most patterns are used as a soft bias only. This allows a bad shape move to still be simulated and chosen by the search.

Section 3.2.1 discusses the learning of patterns. Section 3.2.2 presents the use of patterns in the game tree (defining pattern values, and using pattern values for biasing the tree search). Section 3.2.3 is dedicated to patterns in the playouts (defining values, and using pattern values for biasing the Monte Carlo sampling, out of the tree).

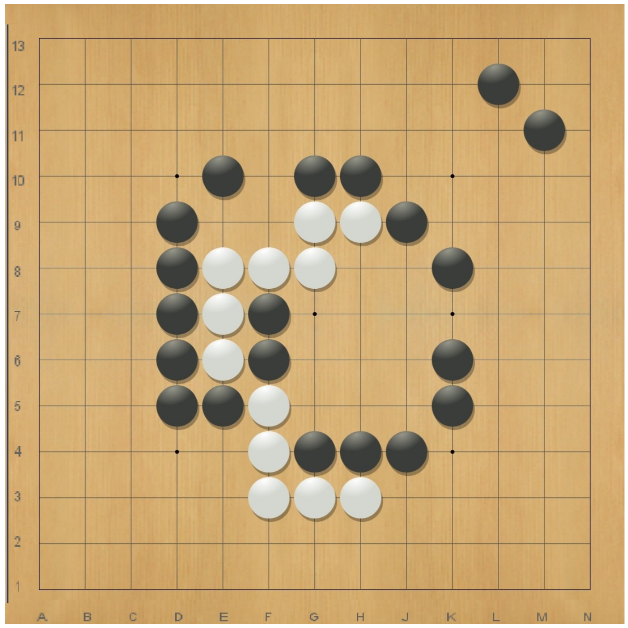



**Fig. 6 Do not play empty triangles (usually):** Playing K11 or L12 for Black builds an empty triangle; it wastes one stone with little effect. **Counter-example:** On the other hand playing G6 for black is a good move; it is the only solution for killing E6.

#### 3.2.1 Patterns Learning

Learning patterns is not an easy task. [19, 14] learn the value of patterns in databases. Let us consider a database of games between strong players. The pattern is a family of pairs (relative location, set of values); for example, an empty triangle pattern (Fig. 6) is "(-1,0) is black, (0,1) is black, (-1,1) is empty, next move is (0,0)". The most popular patterns shapes are approximately circular and rectangular/square. Special case patterns can be learned for corners and sides of the board. Standard corner opening sequences called *joseki* can also be represented as sets of patterns. For matching, symmetries such as rotation, mirroring and swapping colors are taken into account.

The *confidence* of a pattern is the probability (in the database) that a pattern is played, given that it can be played legally. The *support* of a pattern is the frequency at which simultaneously (i) the pattern can legally be played and (ii) it is actually played. There are fast algorithms for selecting patterns which have at least a given support and a given confidence [7]; the speed strongly depends on the support threshold (the lower the threshold, the slower the algorithm).

### 3.2.2 Patterns in the Game Tree

Extracting patterns, with their confidence and support, is the first part of the job; the second part consists of attributing values to patterns.

Heuristic pattern values.

The pattern move value $h(s,m)$, for a move $m$ in a state $s$, can be defined as a combination of pattern size, support, and confidence, using either handcrafted formulas or machine optimization [52]. When the extraction algorithm filters out patterns with low support, then this approach can only design positive patterns, as negative patterns have very low support - Guzumi is therefore neglected.

Minorization-Maximization.

A popular solution for learning patterns values due to Coulom consists in optimizing pattern weights by numerical optimization such as Minorization-Maximization [19, 21]. The criterion for such optimization is typically move prediction in master games.

Direct optimization.

Other approaches directly optimize the winning rate in simulated games [13]. However, this approach requires expensive simulations.

When values are computed, they now have to be used in the program. Heuristic pattern knowledge can be used as a bias in the game tree. A “heuristic pattern value” $h(s,m)$ of each move $m$ in the state $s$ can be added to the MCTS move selection.

Values obtained by reinforcement learning.

Some papers have proposed the use of values obtained by reinforcement learning, but these results were never reproduced later and indeed removed from MoGo in which these tests had been performed.

How to use pattern values.

When patterns are extracted and values are available for all patterns, then the heuristic value of a move $h(s,m)$ can be defined as the value of the largest pattern matching move $m$ in state $s$. Such values can be used for in-tree bias in many different ways. Popular methods are adding “fake” initial simulations for UCT and/or

RAVE values with “virtual wins and losses”, or directly adding a term such as $C \times h(s,m)/\log(n(s,m))$ to the move selection formula [52].

Pattern matching can be based on hashing and be reasonably fast [19]. Values are computed once for a node when it is created. Some implementations maintain two types of nodes; small nodes before computing the pattern value of each move, and large nodes for which values have been computed.

### 3.2.3 Patterns in the Playouts

Using patterns in playouts is important, but computationally much more challenging since typical playouts aim for a speed in the order of one microsecond per move. Besides the still popular small handcrafted patterns such as the original $3 \times 3$ “MoGo patterns” [66] (accompanied by expert rules such as “if you can capture, then capture”, and, for advanced programs, more sophisticated Go expertise such as Nakade and approach moves), we distinguish two learning methods:

- Offline learning: patterns are learnt before the actual game is played.
- Online learning: patterns are learnt during the actual game.

Offline learning of pattern values for Monte Carlo Simulations

An early negative result by Gelly and Silver showed that increasing the strength of a Monte Carlo simulation policy does not necessarily improve the strength of the MCTS player built on top of it [29, 66]. It is more important that a playout is “unbiased” or “balanced”, i.e. that black moves and white moves preserve the equilibrium of the game [61, 35]. Therefore a possible solution consists in defining the balancing problem for $N$ parameters $\theta = (\theta_1, \ldots, \theta_N)$ as follows:

- $V_\theta(s)$: the frequency of wins over $M$ simulations for black when playouts start in situation $s$ and use parameters $\theta$.
- $V_{target}(s)$: a target value, obtained either:
  - by human expertise;
  - or by self-play of a MCTS program on situation $s$.

The case of learning from self-play is particularly interesting because it is unsupervised.

Given $V_{target}(s)$ and a parametric policy defining simulations as a function of a parameter $\theta$, SB provides a parameter $\hat{\theta}$ is obtained by minimization of

$$\hat{\theta} = \arg\min_\theta \sum_{s \in \{s_1,\ldots,s_N\}} (V_\theta(s) - V_{target}(s))^2 . \quad (5)$$

Let us define the random variable *simulation*, which is defined for some initial state $s$ and a parameter $\theta$. The derivative, with respect to $\theta$, of the sum of squares term in Eq. 5 is the product of

$$2\mathbb{E}_s\left(V_\theta(s) - V_{target}(s)\right)$$

(which is easy to compute by simulations, given $\theta$ and $V_{target}$ and

$$\mathbb{E}_{simulation} reward(simulation) \nabla_\theta \log P(simulation|s,\theta),$$

which needs the gradient of the log-likelihood $\log P(simulation|s,\theta)$, which is the sum of the log-likelihood of each move in the simulation.

Success has been reported for simulation balancing on small boards and for the Go program ERICA [34]. However, some negative results were also reported on the computer-go mailing list; sometimes, minimizing Eq. 5 actually decreased the playing strength of the MCTS built on top of the playout.

Despite much research on automated learning methods, handcrafted patterns and/or expert rules are still heavily used in current strong implementations of MCTS for Go. Playouts in which a lot of human knowledge is encoded are termed *heavy playouts*, whereas simulations based on fast simple rules are termed *light playouts*. The recent trend of programs such as ZEN has been towards heavier playouts.

Online learning of pattern values for Monte Carlo simulations.

The RAVE method (Section 3.1) has shown the great potential of exploiting more detailed statistics from simulations. Therefore, it is natural to try other forms of learning from these simulations, possibly leading to dynamic, adaptive policies.

Drake and Chen have proposed to learn a position-specific policy by coevolution of strategies [24]. In spite of promising results on life-and-death situations in Go, such positive results could not be shown for the general case, i.e. for improving the overall strength of optimized MCTS algorithms. Drake and Baier proposed [23, 3] to extract statistics of "winning replies", i.e. moves $b$ for Black which, when following move $w$ for White, have recently led to a win for Black. The early positive results were achieved with relatively simple policies and were not reproduced (at least not in public) for more complex Go programs. The online adaptation of playouts is not, currently, a necessary component of a strong MCTS program.

### *3.3 Expert rules*

Many types of Go knowledge cannot be easily represented by patterns. Examples are capture tactics, irregular cut/connect and life and death problems [46], dealing with wide open spaces [52], threats, and approach moves (moves which prepare a later move, without immediate clear benefit). Such knowledge must, at least for the current state of the art, be encoded manually and needs a lot of handcrafting. Still, such expert rules provide substantial benefits in terms of playing strength of MCTS. Encoding such knowledge into MCTS in the right way is still considered a black art. Parameter tuning methods can help, but surprisingly little has been published on this topic.

### 3.4 Global and local opening books - Fuseki and Joseki

Full board opening books specify the next move given an exactly matching board. They are limited for the full $19 \times 19$ board in Go due to the large branching factor and diversity of possible moves. However, on board sizes of $9 \times 9$ and smaller, books are very powerful. On $7 \times 7$, the MoGo program has shown better than professional performance thanks to its huge opening book built by self-play by a "meta" MCTS algorithm, using a MCTS algorithm as default policy [17]. Importantly, the process still involves some human intervention and trial-and-error.

For big boards ($13 \times 13$ and $19 \times 19$), books can also be used for joseki, local opening sequences. The corners and sides are very important in Go, since it is much easier to build territories and eyes there. "Joseki" refers to standard sequences in a single corner, which are usually but not always played out in the opening phase. In contrast, the term fuseki refers to full board openings, and in part deals with the interaction between different corners and sides. Many current MCTS programs include large-scale patterns, covering josekis and even full board positions. They are usually used for soft pruning, as explained in Section 3.2.2, rather than hardcoded moves.

### 3.5 Parallelizing MCTS for Go

Despite some inherent limitations as discussed above, MCTS generally scales well with increasing time and memory. More simulations and larger trees result in stronger play, with no apparent upper limit in complex games such as Go. To achieve good performance on current hardware such as multicore machines, computer clusters and supercomputers, it is necessary to parallelize MCTS.

Unfortunately, this is a highly nontrivial problem. It is easy to run simulations in parallel. However, the returns from running more simulations on the same node diminish very quickly [72]. The true power of MCTS comes from using the results of previous simulations to grow the tree and choose where future simulations start from, so it is inherently a sequential algorithm. This imposes both theoretical limits and practical challenges to parallelization.

The two main types of parallel MCTS are differentiated by the memory model. Multi-core machines such as workstations provide a single shared memory, while clusters with distributed memory architecture require an approach using message-passing. On large-scale modern machines with fast networks and a multi-level memory hierarchy this formerly clear distinction begins to blur. We discuss the main algorithms for different architectures, then present theoretical scalability issues which limit the efficiency of parallelization.

Besides the works discussed in more detail below, other important publications on parallel MCTS in Go include [11, 15, 37, 32].

### 3.5.1 Shared Memory Parallelization on a Multi-core Machine

Parallelizing MCTS on a multi-core machine is usually performed by running the loop of MCTS (Section 2.2) simultaneously on each core, with each core updating the same game tree in memory. The two main limitations are memory contention and diversity of simulations. Memory contention occurs whenever different threads access the same memory location representing the same node. Increasing the number of threads also puts strain on the memory bus systems. Locking the tree for each update is not a realistic option - lock-free algorithms are required [20, 26]. The other risk is that without global control, all cores will run very similar simulations. The technique of virtual loss [15] temporarily adds a loss to the tree before the simulation is run. This encourages other threads to explore different parts of the tree. If the simulation turns out to be a win, the virtual loss is changed accordingly. Some authors have also experimented with virtual wins.

### 3.5.2 Parallelization with message passing

On message-passing machines such as off-the-shelf clusters, it is too slow to share memory directly. A classical solution [11, 5] used in MoGo works as follows:

- A separate MCTS runs on each machine.
- With low frequency, such as three times per second, all nodes broadcast statistics such as number of simulations, number of wins, number of RAVE simulations, and number of RAVE wins for all “heavy” nodes which have been included in at least some percentage of the total number of simulations. Such broadcast can be performed in time logarithmic in the number of machines if the network structure supports it.
- Each node augments its statistics using these messages and keeps running.

### 3.5.3 Parallelization with Distributed Tree and Depth-First UCT

One severe disadvantage of the solution above is the large amount of duplication. Many trees are built concurrently, each starting from the root, and due to the partial synchronisation there is a large amount of overlap between them. It is therefore tempting to develop methods where a single tree is distributed over all nodes in a parallel system. UCT-Treesplit is one such approach, used in the program GoMorra [32]. Another major bottleneck to parallelization is that in the classical formulation of MCTS, each simulation result is propagated all the way to the root node.

This causes heavy memory contention near the root, which limits speedup. Depth-first UCT [73] is a reformulation of UCT which drastically reduces the number of such updates at only a small loss in precision of in-tree move selection. It is used in conjunction with a distributed transposition table in MPFUEGO, the massively parallel version of FUEGO.

#### 3.5.4 Limits to Scalability

Each parallel MCTS algorithm suffers from a loss of information compared to a sequential one, since the information about the result of the concurrently running simulations is not available for decision-making. Segal investigates the theoretical limit of parallel MCTS and concludes that current programs are still very far from reaching that limit [58]. Sections 5.1 and 5.2 discuss several situations for which increasing the computation time within reasonable limits does not help for current Go programs. Many programs suffer from limits in their scalability as a consequence. For the case of MOGO, this is shown in Table 1.

| $N$ =Number of simulations | Success rate of 2N simulations against $N$ simulations in $9 \times 9$ Go | Success rate of 2N simulations against $N$ simulations in $19 \times 19$ Go |
|---|---|---|
| 1 000 | $71.1 \pm 0.1$ % | $90.5 \pm 0.3$ % |
| 4 000 | $68.7 \pm 0.2$ % | $84.5 \pm 0.3$ % |
| 16 000 | $66.5 \pm 0.9$ % | $80.2 \pm 0.4$ % |
| 256 000 | $61.0 \pm 0.2$ % | $58.5 \pm 1.7$ % |

**Table 1** Scalability of MCTS for MOGO. These results show a decrease of scalability as computational power increases.

# 4 The Current Performance of MCTS Go Programs

## *4.1 A Brief History of Computer Go Since The Beginning of MCTS*

MCTS was born in 2006. Since that time, it has completely dominated the computer Go scene.

- The UEC Cup is the largest tournament for $19 \times 19$ Go and has been held since 2007. It has always been won by MCTS programs: CRAZY STONE in 2007, 2008 and 2013; KCC IGO in 2009; FUEGO in 2010; and ZEN in 2011.
- Since 2007, all Computer Olympiads in $19 \times 19$ Go have been won by MCTS programs: in 2007 MOGO; 2008 the MCTS version of THE MANY FACES OF GO; 2009, 2011 and 2013 ZEN; 2010 ERICA.

- On $9 \times 9$, MCTS programs have won every Olympiad since 2006 with CRAZY STONE, followed by STEENVRETER in 2007, THE MANY FACES OF GO in 2008, FUEGO in 2009, MYGOFRIEND in 2010, and ZEN in 2011 and 2013.
- A $13 \times 13$ competition has been held since 2010 and all tournaments were won by MCTS programs: THE MANY FACES OF GO in 2010, and ZEN in 2011 and 2013.

### *4.2 Performance against Professional Human Players*

As of the end of 2013, computer programs are on a par with the top human players on $9 \times 9$, and about 4 handicap stones weaker on the big $19 \times 19$ board. Figures 8 and 7 summarize the most important wins against humans in $9 \times 9$ and $19 \times 19$ Go respectively [67]. The first significant wins in even games were achieved in $9 \times 9$ Go. The first such win against a pro was achieved by MOGO in 2008, the first win against a world champion-level 9P was by FUEGO in 2009, and the first such win with the disadvantageous side (black) was by MOGO later in 2009.

However, despite a considerable number of further wins, even in 2013 there is still no proven superiority of computers against the best humans in $9 \times 9$ Go. A recent negative result was when the top program ZEN was beaten in six straight games by human professionals So Yokoku 8P, Ohashi Hirofumi 5P and Ichiriki Ryo 2P in November 2012.

In $19 \times 19$ Go, the best current programs CRAZY STONE and ZEN are ranked as 6 Dan amateur, the highest “normal” amateur rank. As of November 24th, 2013, a version of ZEN is playing on KGS (Kiseido Go Server) holds a stable 6 dan rank. The difference to a top professional is about 4 ranks, corresponding to 4 handicap stones. This is confirmed by recent demonstration games [67].

## 5 Limitations of Current Go Programs

This section describes cases in which MCTS usually fails in Go:

- Situations in which many symmetries are involved (Section 5.1);
- Situations in which several local fights interact, leading to a complex strategic problem (Section 5.2);
- Situations in which the game requires the player to think about a distant move (Section 5.3);
- Situations which involve long term effects (Section 5.4).

In such situations, increasing the computational power does not improve the outcome, and MCTS has weak scalability of MCTS. Therefore, parallelization as in Section 3.5 is not sufficient to overcome these problems.

| Date | Human player & rank | Handicap | Program |
|---|---|---|---|
| 2008/8 | Kim Myungwan 8P | 9 stones | MoGo |
| 2008/9 | Kaori Aoba 4P | 8 stones | Crazy Stone |
| 2008/12 | Kaori Aoba 4P | 7 stones | Crazy Stone |
| 2009/2 | Li-Chen Chien 1P | 6 stones | MoGo |
| 2009/2 | **Chun-Hsun Chou** 9P | 7 stones | **MoGo** |
| 2009/3 | James Kerwin 1P | 6 stones | Many Faces of Go |
| 2010/11 | Kaori Aoba 4P | 5 stones | Zen |
| 2011/8 | Kozo Hayashi 6P | 5 stones | Zen |
| 2011/12 | Meiko Tei 9P | 6 stones | Zen |
| 2012/3 | **Masaki Takemiya 9P** | 5 stones | **Zen** |
| 2012/3 | **Masaki Takemiya 9P** | 4 stones | **Zen** |
| 2012/7 | **Chun-Hsun Chou 9P** | 4 stones | **Zen** |
| 2012/8 | Catalin Taranu 5P | 4 stones | Crazy Stone |
| 2012/11 | **Chun-Hsun Chou 9P** | 4 stones | **Zen** |
| 2013/3 | **Yoshio Ishida 9P** | 4 stones | **Crazy Stone** |

**Fig. 7** Selected computer wins against professionals in $19 \times 19$ Go. Only wins are shown, even when they were accompanied by losses (e.g. one win out of three games). Wins against former world championship winners in bold.

| Date | Human player & rank | Program |
|---|---|---|
| 2008/3 | Catalin Taranu 5p | MoGo |
| 2009/2 | Shih Chin 2p | MoGo |
| 2009/5 | Catalin Taranu 5p | MoGo |
| 2009/8 | **Chun-Hsun Chou 9P** | **Fuego** |
| 2009/8 | Fan Hui 2p | MoGo |
| 2009/10 | **Chun-Hsun Chou 9P** | **MoGo** |
| 2010/1 | Chi-hyeuong Nam 1p | Zen |

**Fig. 8** Selected computer wins against professionals in $9 \times 9$ Go. Only even games with no handicap and estimated fair Komi of 6.5, 7 or 7.5 are shown.

## *5.1 Exploiting Symmetry*

Humans excel at visual perception, including recognizing and understanding symmetries. This is true for reading; humans can easily read mirrored characters, characters written with reversed colors, as well as randomly rotated characters. Maybe more importantly, humans can also understand partial symmetries.

In terms of Go, humans do not waste time analyzing equivalent moves separately. Large numbers of equivalent moves, or move sequences, can lead to combinatorial explosion, and the wasted time becomes huge. A critical case in Go are capturing races called *semeai*. In semeai, many move sequences which reduce the liberties of opponent blocks are exactly equivalent. A MCTS algorithm will waste time trying to prove which of these equivalent moves is best, and reach only shallow depths, while humans use very deep, focused reading up to capture or up to a standard situation that can be evaluated statically. Simulation policies usually do not have enough knowledge to resolve all but the simplest semeai. Learned patterns also do

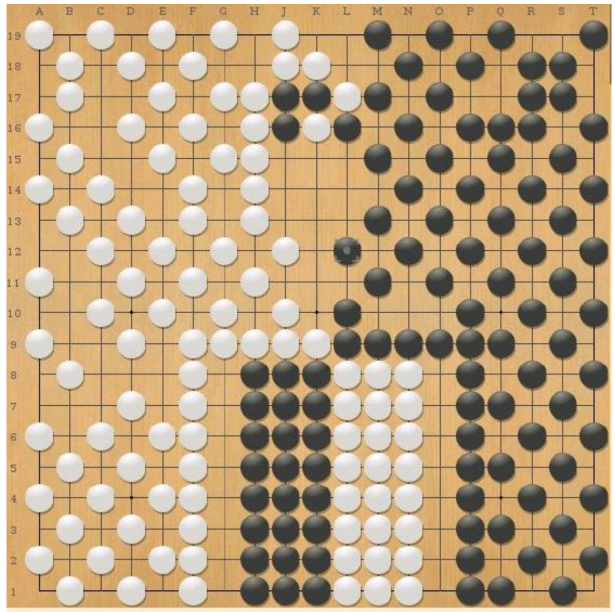


**Fig. 9** White to play. In the 8 vs 8 liberty semeai at the bottom, White must immediately attack the black group J4 by taking one of its liberties. This also saves M4.

not help since the liberty-filling moves are usually not good shape and have no other meaning than to kill.

As a consequence, no current MCTS program can distinguish reliably between situations like Fig. 9, which is a semeai which must be played immediately, and Fig. 10, a semeai which has been decided.

## *5.2 Local Search for Divide and Conquer*

MCTS has provided great improvements in the general fighting ability of Go programs. However, they have trouble handling multiple simultaneous fights. Huang and Müller study two such cases, termed "two safe groups" and seki (coexistence) [36]. They develop a test suite of such problems and test it against most of the current top programs. These problems contain simple local fights, but are too deep to be jointly resolved by global search. Only very few programs have enough knowledge in their simulation policy to do well. A famous example of a professional game where two local fights interact in a complex way is shown in Fig. 11.

Divide and conquer approaches have been successful for endgame puzzles [44] and for small isolated life-and-death problems [38], but not yet in the context of a full board MCTS program. Sheppard [59] proposed the use of MCTS with macro-

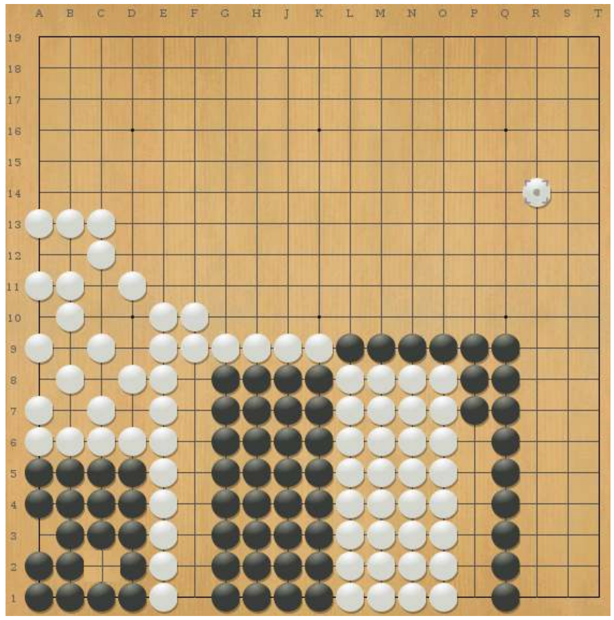


**Fig. 10** Black to play is ahead by 8 vs 6 liberties and should not play in the semeai - it would be a wasted move.

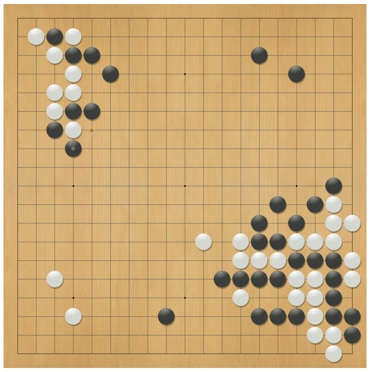
actions. This looks reasonable, but as far as we know has not yet been successfully implemented.

## *5.3 The Art of Tenuki*

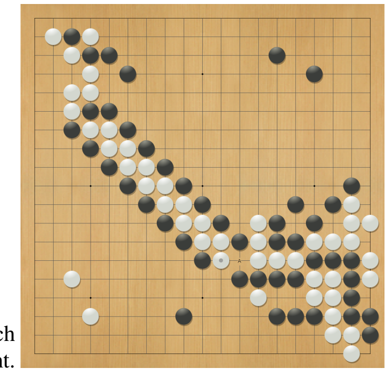
One of the simplest expert rules used in MCTS algorithms is locality: very often, the best move is close to the previous one. Deciding when to answer locally, and when to “tenuki”, i.e. switch to elsewhere on the board, is considered as a key competence for playing Go. In terms of combinatorial game theory, it amounts to evaluating and comparing the temperature - urgency of playing in - different parts of the board. Patterns and human expertise are good for helping MCTS in soft pruning, hence partially counterbalance the bonus for local play.

An illustrative proverb says “If it’s worth only 15 points, play tenuki”. MCTS works by simulations and wins and losses; the concept of points is not included in MCTS.

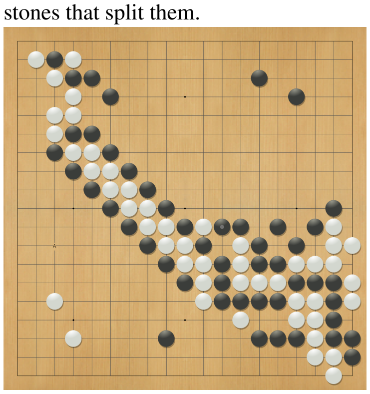
One computational inefficiency of the full-board, global search used in MCTS is that it needs to spend computational effort on a candidate tenuki move each time while playing out a local sequence. It cannot re-use computations about other parts of the board performed during earlier moves, since they are all embedded in an ever-changing global context. In many cases, the value of a move can be computed



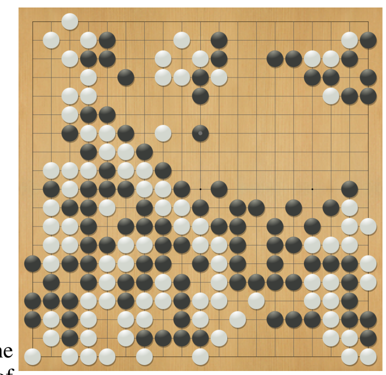
Black starts a ladder in the top left, which will have an impact on the bottom right. In that corner, a large-scale joseki has ended peacefully. The three white groups have won a semeai against the eight black stones that split them.



The ladder attack fails, as even a weak Go player can read out.



Black played the bad ladder on purpose: he uses forcing moves to reverse the state of the semeai, kill all the white stones in the bottom right, and also builds strong influence towards the top right.



The game at a later stage. Black has a huge territory on the right side and wins.

**Fig. 11** Lee Sedol (black) plays against Hong Jansik (2003, April 23rd).

(mostly) locally. Humans can estimate such a local move value and switch at the appropriate time, recalculating only if the global context changes. While specialized programs can deal efficiently with the case where local situations are *completely* separate, humans can handle the much more frequent case of several approximately independent fights on the same Go board.

### *5.4 Long-Term Effects*

Some situations require subtle moves which have a long-term impact. Similarly, moves can be bad because of a long-term negative effect. Long-term effects are difficult for MCTS since they can not be handled in the tree - they are only resolved in the (noisy, low quality) simulation phase. Avoiding moves with such bad long-term effects is a major open problem for MCTS in Go, and is discussed in some detail in this section.

MCTS simulations consist of two parts:

- an in-tree part containing (reasonably) good moves;
- Monte Carlo simulations or playouts, which are randomized and contain many blunders.

As a consequence, later parts of the game are very poorly simulated. This is actually much improved when *heavy* playouts are used. Yet, the quality is much better for in-tree moves. This implies that a move which has bad long term consequences is not reliably evaluated as bad by the simulations. Besides the cases of two safe groups and seki mentioned in Section 5.2, Ko fights are a classic example. In Figure 12, White can save the group at the top late in the game because White has a Ko threat at the bottom. However, most MCTS programs would have played the forcing exchange F2 for F1 much earlier in the game, and as a consequence they would be out of Ko threats at this stage. Therefore, it is important to not to waste threats, and keep them for Ko fights that might materialize later. Most MCTS programs are not able to do that, for (at least) two reasons. One problem is that locally, F2 is a good move, gaining one point in sente. However, it is more important to keep it in reserve as a threat until late in the endgame. Another problem is specific to MCTS with its evaluation of a node as the weighted average of its children. A threat which has only one good reply will have a too-high value, since it inherits some of the high evaluations for all the other branches where the opponent ignored the threat. In the example, if White plays F2 and Black answers anywhere else but F1, then all these positions will be big wins for White.

## 6 Conclusions

Currently all strong Go programs are based on MCTS. In the early days of MCTS for Go, it was possible to design such a program without any expertise in Go, using just light playouts. However, introducing human expertise is now critical for reaching a competitive playing level; all strong programs use such knowledge in the playouts and in the tree. This knowledge can be partially learnt offline, automatically, on databases of games; and online, thanks to RAVE values. Also results can be improved by the parallelization on multi-core machines or clusters. Unfortunately, as with classical programs, the key bottleneck remains the encoding of human exper-

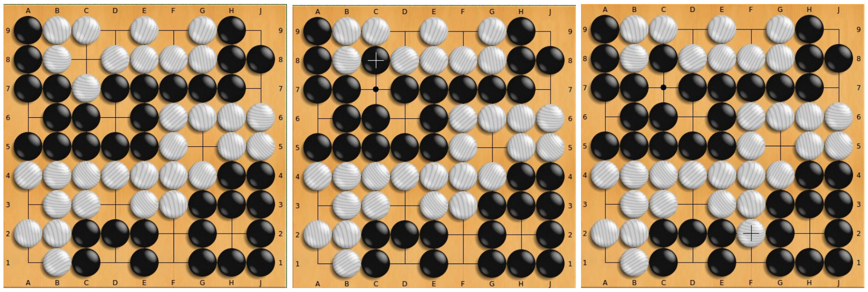


**Fig. 12 The importance of keeping Ko threats.** Left: Black captures the Ko with C8, leading to the middle position. Middle: White can not immediately recapture at C7 due to the Ko rule, but White can play a Ko threat at F2, leading to the position on the right. Right: if Black ignores the threat, White will play F1 next and kill the bottom side. If Black plays F1, White can now recapture at C7. Black has no threats, so White can also connect at C8 and live. Thanks to the threat at F2, White won the Ko fight and lived.

tise, despite progress in automated methods such as large-scale learning of pattern weights.

### *Future Work and Open Problems*

There are many clearly defined situations which are poorly handled by MCTS Go programs, such as divide and conquer problems, semeai, and long term effects such as Ko threats. While a lot of research has been devoted to these problems, including online or offline learning, many challenges still remain unsolved.

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## References

1. P. Auer, N. Cesa-Bianchi, and P. Fischer. Finite time analysis of the multiarmed bandit problem. *Machine Learning*, 47(2/3):235–256, 2002.
2. P. Auer, N. Cesa-Bianchi, Y. Freund, and R. E. Schapire. Gambling in a rigged casino: the adversarial multi-armed bandit problem. In *Proceedings of the 36th Annual Symposium on Foundations of Computer Science*, pages 322–331. IEEE Computer Society Press, Los Alamitos, CA, 1995.
3. H. Baier and P. Drake. The power of forgetting: Improving the last-good-reply policy in Monte Carlo Go. *IEEE Transactions on Computational Intelligence and AI in Games*, 2(4):303–309, 2010.

4. D. Benson. Life in the game of Go. *Information Sciences*, 10:17–29, 1976. Reprinted in Computer Games, Levy, D.N.L. (Editor), Vol. II, pp. 203-213, Springer Verlag, New York 1988.
5. A. Bourki, G. Chaslot, M. Coulm, V. Danjean, H. Doghmen, J.-B. Hoock, T. Hérault, A. Rimmel, F. Teytaud, O. Teytaud, P. Vayssière, and Z. Yu. Scalability and parallelization of monte-carlo tree search. In *Proceedings of Advances in Computer Games 13*, 2010.
6. B. Bouzy and T. Cazenave. Computer Go: An AI-oriented survey. *Artificial Intelligence*, 132(1):39–103, 2001.
7. B. Bouzy and G. Chaslot. Bayesian generation and integration of k-nearest-neighbor patterns for 19x19 go. *In G. Kendall and Simon Lucas, editors, IEEE 2005 Symposium on Computational Intelligence in Games, Colchester, UK*, pages 176–181, 2005.
8. R. Bozulich. *The Go Players Almanac 2001*. Kiseido, Tokyo, 2001.
9. C. Browne, E. Powley, D. Whitehouse, S. Lucas, P. Cowling, P. Rohlfshagen, S. Tavener, D. Perez, S. Samothrakis, and S. Colton. A survey of Monte Carlo tree search methods. *IEEE Trans. Comput. Intellig. and AI in Games*, 4(1):1–43, 2012.
10. B. Bruegmann. Monte-carlo Go, 1993. Unpublished draft, `http://www.althofer.de/Bruegmann-MonteCarloGo.pdf`.
11. T. Cazenave and N. Jouandeau. On the parallelization of UCT. In *Proceedings of CGW07*, pages 93–101, 2007.
12. G. Chaslot, S. Bakkes, I. Szita, and P. Spronck. Monte Carlo Tree Search: A New Framework for Game AI. In C. Darken and M. Mateas, editors, *AIIDE*. The AAAI Press, 2008.
13. G. Chaslot, M. Winands, I.Szita, and H. van den Herik. Parameter tuning by cross entropy method. In *European Workshop on Reinforcement Learning*, 2008.
14. G. Chaslot, M. Winands, J. Uiterwijk, H. van den Herik, and B. Bouzy. Progressive Strategies for Monte-Carlo Tree Search. In P. Wang et al., editors, *Proceedings of the 10th Joint Conference on Information Sciences (JCIS 2007)*, pages 655–661. World Scientific Publishing Co. Pte. Ltd., 2007.
15. G. M. J.-B. Chaslot, M. H. M. Winands, and H. J. van den Herik. Parallel Monte-Carlo Tree Search. In *Proc. Comput. and Games, LNCS 5131*, pages 60–71, Beijing, China, 2008.
16. B. E. Childs, J. H. Brodeur, and L. Kocsis. Transpositions and move groups in Monte Carlo Tree Search. In P. Hingston and L. Barone, editors, *IEEE Symposium on Computational Intelligence and Games*, pages 389–395. IEEE, 2008.
17. C.-W. Chou, P.-C. Chou, H. Doghmen, C.-S. Lee, T.-C. Su, F. Teytaud, and O. Teytaud. Towards a solution of 7x7 Go with Meta-MCTS. In *Proceedings of Advances in Computer Games 2011*, pages November 20–23, 2011, Tilburg University, The Netherlands, 2011.
18. R. Coulom. Efficient Selectivity and Backup Operators in Monte-Carlo Tree Search. *In P. Ciancarini and H. J. van den Herik, editors, Proceedings of the 5th International Conference on Computers and Games, Turin, Italy*, 2006.
19. R. Coulom. Computing elo ratings of move patterns in the game of go. In *Computer Games Workshop, Amsterdam, The Netherlands*, 2007.
20. R. Coulom. Lockless hash table and other parallel search ideas, 2008. `http://www.mail-archive.com/computer-go@dvandva.org/msg07611.html`.
21. R. Coulom. Clop: Confident local optimization for noisy black-box parameter tuning. In *Advances in Computer Games*, pages 146–157. Springer Berlin Heidelberg, 2012.
22. M. Crasmaru and J. Tromp. Ladders are PSPACE-complete. In *Computers and Games*, pages 241–249, 2000.
23. P. Drake. The last-good-reply policy for monte-carlo go. *ICGA Journal*, 32(4):221–227, 2009.
24. P. Drake and Y.-P. Chen. Coevolving partial strategies for the game of go. In *International Conference on Genetic and Evolutionary Methods*. CSREA Press, 2008.
25. M. Enzenberger. Evaluation in go by a neural network using soft segmentation. *10th Advances in Computer Games Conference, Graz*, 2003.
26. M. Enzenberger and M. Müller. A lock-free multithreaded Monte-Carlo tree search algorithm. In J. van den Herik and P. Spronck, editors, *Advances in Computer Games*, volume 6048 of *Lecture Notes in Computer Science*, pages 14–20, Pamplona, Spain, 2010. Springer Verlag.

27. S. Flory and O. Teytaud. Upper confidence trees with short term partial information. In *Procedings of EvoGames 2011*, pages 153–162. Springer, 2011.
28. Free Software Foundation. GNU Go, 1989-2013. Date retrieved: Nov 18, 2013. `http://www.gnu.org/software/gnugo/`.
29. S. Gelly. *Une contribution à l'apprendissage par renforcement ; application au Computer GO*. PhD thesis, Université Paris-Sud - Paris XI, September 2007. Paris Universities Award and 2nd Prize at Gilles Kahn - Académie des Sciences Awards.
30. S. Gelly and D. Silver. Combining online and offline knowledge in UCT. In *ICML '07: Proceedings of the 24th international conference on Machine learning*, pages 273–280, New York, NY, USA, 2007. ACM Press.
31. S. Gelly and D. Silver. Monte-carlo tree search and rapid action value estimation in computer go. *Artif. Intell.*, 175(11):1856–1875, July 2011.
32. T. Graf, U. Lorenz, M. Platzner, and L. Schaefers. Parallel Monte-Carlo Tree Search for HPC Systems. In *Proc. 17th Int. Euro. Conf. Parallel Distrib. Comput., LNCS 6853*, pages 365–376, Bordeaux, France, 2011.
33. M. D. Grigoriadis and L. G. Khachiyan. A sublinear-time randomized approximation algorithm for matrix games. *Operations Research Letters*, 18(2):53–58, Sep 1995.
34. S. Huang. *New Heuristics for Monte Carlo Tree Search Applied to the Game of Go*. PhD thesis, National Taiwan Normal University, 2011.
35. S.-C. Huang, R. Coulom, and S.-S. Lin. Monte-carlo simulation balancing in practice. In H. J. van den Herik, H. Iida, and A. Plaat, editors, *Computers and Games*, volume 6515 of *Lecture Notes in Computer Science*, pages 81–92. Springer, 2010.
36. S.-C. Huang and M. Müller. Investigating the limits of Monte Carlo tree search methods in computer Go, 2013. Accepted for Computers and Games 2013. 12 pp.
37. H. Kato and I. Takeuchi. Parallel Monte-Carlo Tree Search with Simulation Servers. In *Proc. Int. Conf. Tech. Applicat. Artif. Intell.*, pages 491–498, Hsinchu City, Taiwan, 2010.
38. A. Kishimoto and M. Müller. Dynamic decomposition search: A divide and conquer approach and its application to the one-eye problem in go. In *IEEE Symposium on Computational Intelligence and Games (CIG'05)*, pages 164–170, 2005.
39. A. Kishimoto and M. Müller. Search versus knowledge for solving life and death problems in Go. In *Twentieth National Conference on Artificial Intelligence (AAAI-05)*, pages 1374–1379, 2005.
40. L. Kocsis and C. Szepesvari. Bandit based Monte-Carlo planning. In *15th European Conference on Machine Learning (ECML)*, pages 282–293, 2006.
41. L. Kocsis and C. Szepesvari. Discounted-UCB. In *2nd Pascal-Challenge Workshop*, 2006.
42. T. Lai and H. Robbins. Asymptotically efficient adaptive allocation rules. *Advances in Applied Mathematics*, 6:4–22, 1985.
43. M. Müller. Playing it safe: Recognizing secure territories in computer Go by using static rules and search. In H. Matsubara, editor, *Game Programming Workshop in Japan '97*, pages 80–86, Computer Shogi Association, Tokyo, Japan, 1997.
44. M. Müller. Decomposition search: A combinatorial games approach to game tree search, with applications to solving Go endgames. In *Sixteenth International Joint Conference on Artificial Intelligence (IJCAI-99)*, pages 578–583, Stockholm, Sweden, 1999.
45. M. Müller. Review: Computer Go 1984 - 2000. In T. Marsland and I. Frank, editors, *Computers and Games 2000*, number 2063 in Lecture Notes in Computer Science, pages 426–435. Springer Verlag, 2001.
46. M. Müller. Computer Go. *Artificial Intelligence*, 134(1–2):145–179, 2002.
47. M. Müller. Rave problems, 2009. `http://www.mail-archive.com/computer-go@dvandva.org/msg12189.html`.
48. X. Niu, A. Kishimoto, and M. Müller. Recognizing seki in computer Go. In J. van den Herik, S.-C. Hsu, T.-s. Hsu, and H. Donkers, editors, *Advances in Computer Games*, volume 4250 of *Lecture Notes in Computer Science*, pages 88 – 103. Springer, 2006.
49. X. Niu and M. Müller. An open boundary safety-of-territory solver for the game of Go. In J. van den Herik, P. Ciancarini, and H. Donkers, editors, *Computers and Games*, volume 4630 of *Lecture Notes in Computer Science*, pages 37 – 49, Torino, Italy, 2007. Springer.

50. X. Niu and M. Müller. An improved safety solver in Go using partial regions. In J. van den Herik, X. Xu, Z. Ma, and M. Winands, editors, *Computers and Games*, volume 5131 of *Lecture Notes in Computer Science*, pages 102–112, Beijing, China, 2008. Springer.
51. W. Reitman and B. Wilcox. The structure and performance of the Interim.2 Go program. In *IJCAI-79*, pages 711–719, 1979.
52. A. Rimmel, O. Teytaud, C.-S. Lee, S.-J. Yen, M.-H. Wang, and S.-R. Tsai. Current Frontiers in Computer Go. *IEEE Transactions on Computational Intelligence and Artificial Intelligence in Games*, page in press, 2010.
53. B. Robertie. Backgammon. *Inside Backgammon*, 2(1):4, 1980.
54. J. M. Robson. The complexity of go. In *IFIP Congress*, pages 413–417, 1983.
55. A. Saffidine, T. Cazenave, and J. Méhat. UCD : Upper confidence bound for rooted directed acyclic graphs. *Knowl.-Based Syst.*, 34:26–33, 2012.
56. J. Schaeffer. The history heuristic and alphavbeta search enhancements in practice. *IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI-11*, pages 1203–1212, 1989.
57. N. N. Schraudolph, P. Dayan, and T. J. Sejnowski. Temporal difference learning of position evaluation in the game of go. In J. D. Cowan, G. Tesauro, and J. Alspector, editors, *NIPS*, pages 817–824. Morgan Kaufmann, 1993.
58. R. Segal. On the scalability of parallel UCT. In H. van den Herik, H. Iida, and A. Plaat, editors, *Computers and Games*, volume 6515 of *Lecture Notes in Computer Science*, pages 36–47. Springer Berlin / Heidelberg, 2011.
59. B. Sheppard. A pro-game which is computer-unreadable, 2011. `http://www.mail-archive.com/computer-go@dvandva.org/msg04207.html`.
60. D. Silver, R. Sutton, and M. Müller. Sample-based learning and search with permanent and transient memories. In A. McCallum and S. Roweis, editors, *Proceedings of the 25th International Conference on Machine Learning*, pages 968–975. Omnipress, 2008.
61. D. Silver and G. Tesauro. Monte-carlo simulation balancing. In *Proceedings of the 26th Annual International Conference on Machine Learning*, ICML '09, pages 945–952, New York, NY, USA, 2009. ACM.
62. R. Sutton and A. Barto. *Reinforcement learning: An introduction.* MIT Press., Cambridge, MA, 1998.
63. T. Thomsen. Lambda-search in game trees - with application to Go. *ICGA Journal*, 23(4):203–217, 2000.
64. E. van der Werf, H. van den Herik, and J. Uiterwijk. Solving Go on small boards. *ICGA Journal*, 26(2):92–107, 2003.
65. E. van der Werf and M. Winands. Solving Go for rectangular boards. *ICGA Journal*, 32(2):77–88, 2009.
66. Y. Wang and S. Gelly. Modifications of UCT and sequence-like simulations for Monte-Carlo Go. In *IEEE Symposium on Computational Intelligence and Games, Honolulu, Hawaii*, pages 175–182, 2007.
67. N. Wedd. Human-computer Go challenges, 2013. Date retrieved: Nov 18, 2013. `http://www.computer-go.info/h-c/index.html`.
68. Wikipedia. Go (game), 2013. [Online: `http://en.wikipedia.org/wiki/Go_(board_game)`; accessed Nov 29, 2013].
69. Wikipedia. Rules of go, 2013. [Online: `http://en.wikipedia.org/wiki/Rules_of_Go`; accessed Nov 29, 2013].
70. B. Wilcox. Computer Go. In D. Levy, editor, *Computer Games*, volume 2, pages 94–135. Springer-Verlag, 1988.
71. T. Wolf. Forward pruning and other heuristic search techniques in tsume go. *Information Sciences*, 122:59–76, 2000.
72. H. Yoshimoto, K. Yoshizoe, T. Kaneko, A. Kishimoto, and K. Taura. Monte Carlo Go has a way to go. In *AAAI*, pages 1070–1075. AAAI Press, 2006.
73. K. Yoshizoe, A. Kishimoto, T. Kaneko, H. Yoshimoto, and Y. Ishikawa. Scalable distributed Monte-Carlo tree search. In D. Borrajo, M. Likhachev, and C. L. López, editors, *Symposium on Combinatorial Search*, pages 180–187. AAAI Press, 2011.
74. A. L. Zobrist. A hashing method with applications for game playing. Technical Report 88, Computer Sciences Department, University of Wisconsin, 1970.